Atmospheric Optical Turbulence Characterization for the Airborne Laser Using Combined Measurement and Simulation Techniques

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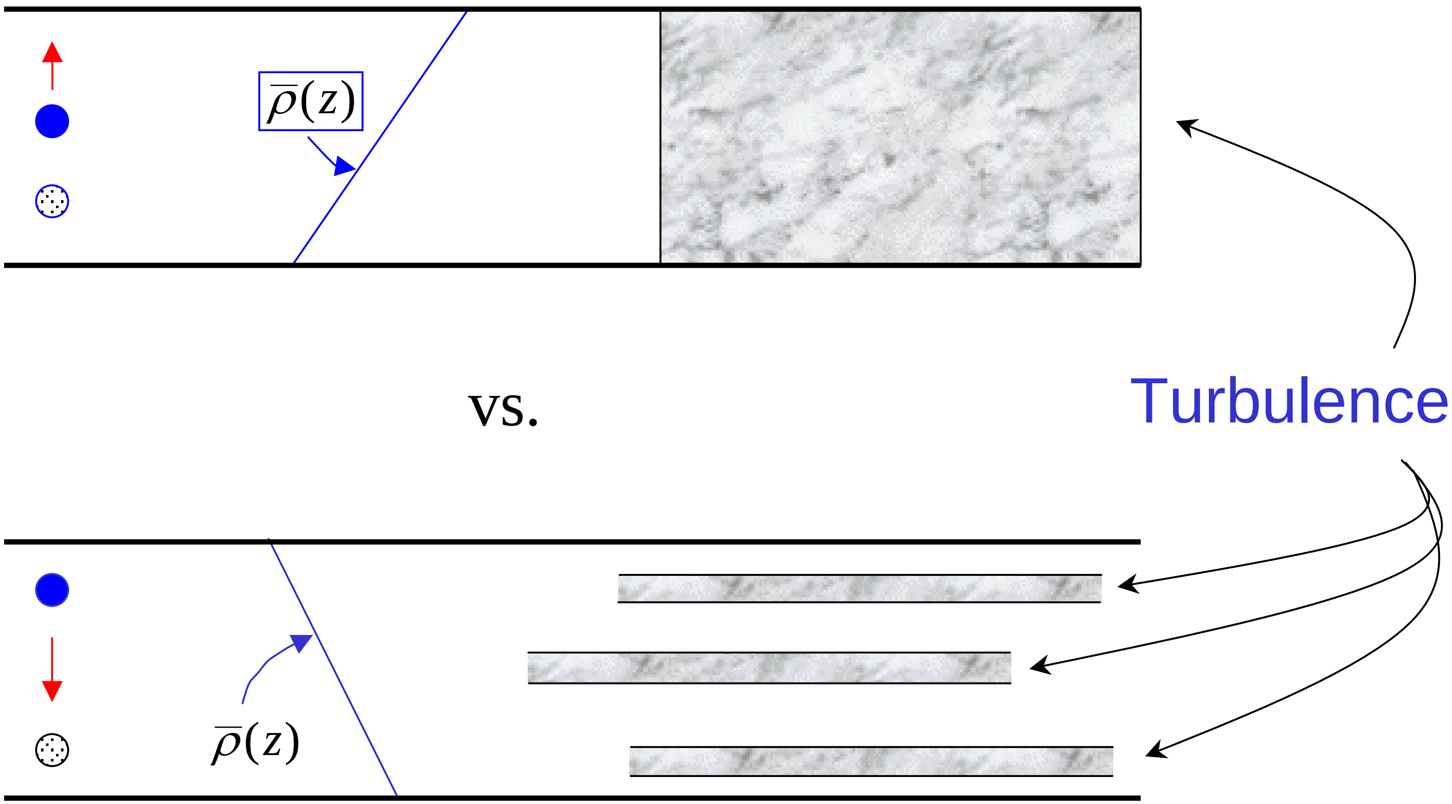
ERDC MSRC

T3E and archival storage resources:
ERDC and NAVO

Overview of our ABL-Component Objectives

- Use numerical simulations to improve/augment atmospheric turbulence characterization so that:
  - phase-screen specification for optical propagation simulations may be evaluated and possibly improved, and
  - more intelligent modeling may proceed, admitting simulation of larger-scale processes and the development of a reliable atmospheric decision aid.

The Problem with Stratified Fluids



Challenges for ABL Simulation

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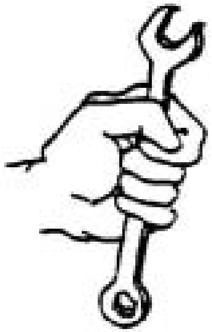
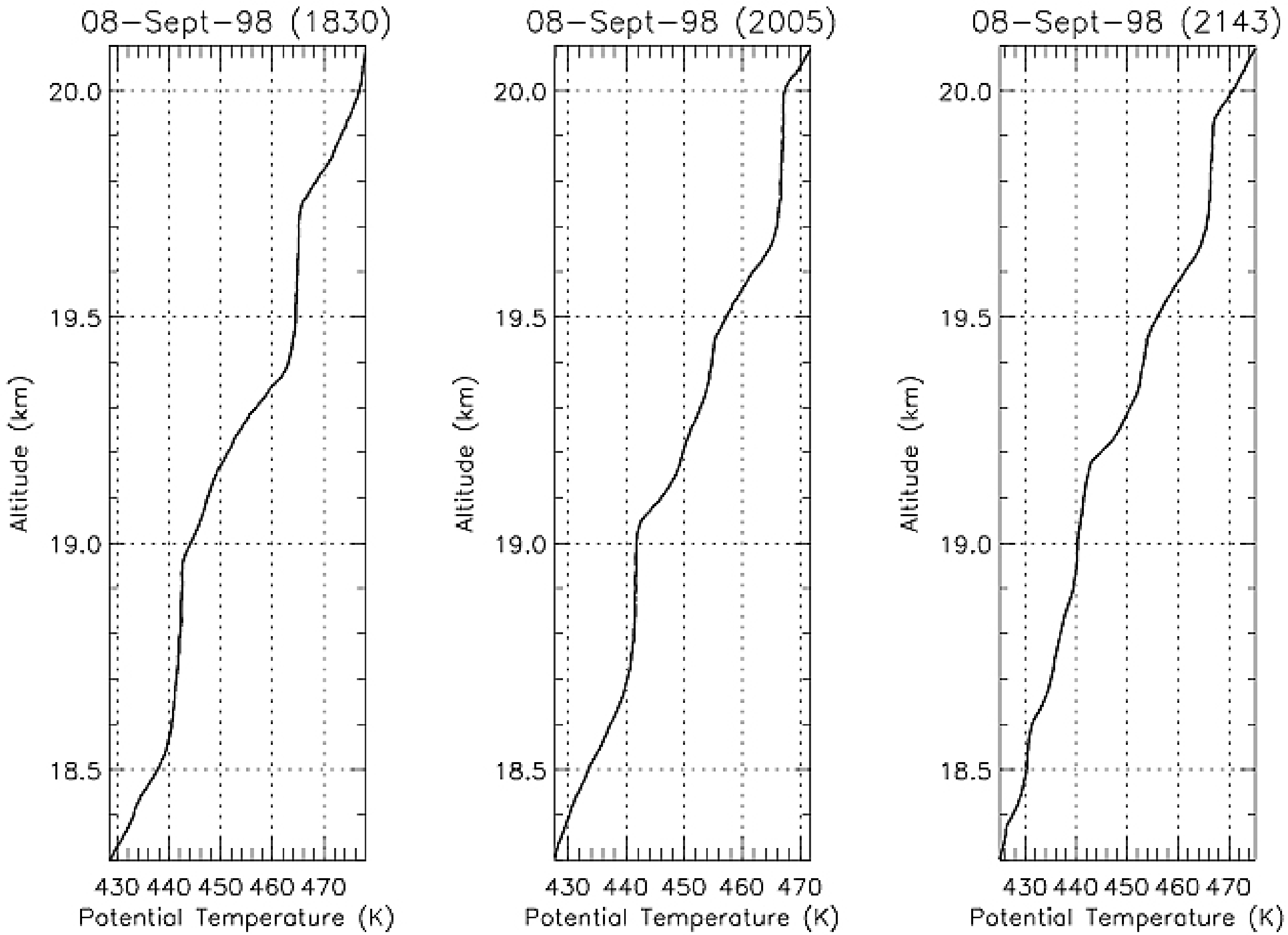
- Range of scales: from 200km to 100m (or smaller).
- Non-Kolmogorov: current sub-grid-scale (SGS) turbulence parameterization schemes are inadequate for stable stratification.
- Combined numerical/observational studies are feasible for developing improved phase-screen descriptions, but simulations of isolated turbulent layers still require state-of-the-art techniques.

Potential-Temperature Steps in the Stratosphere

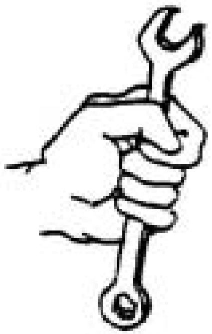


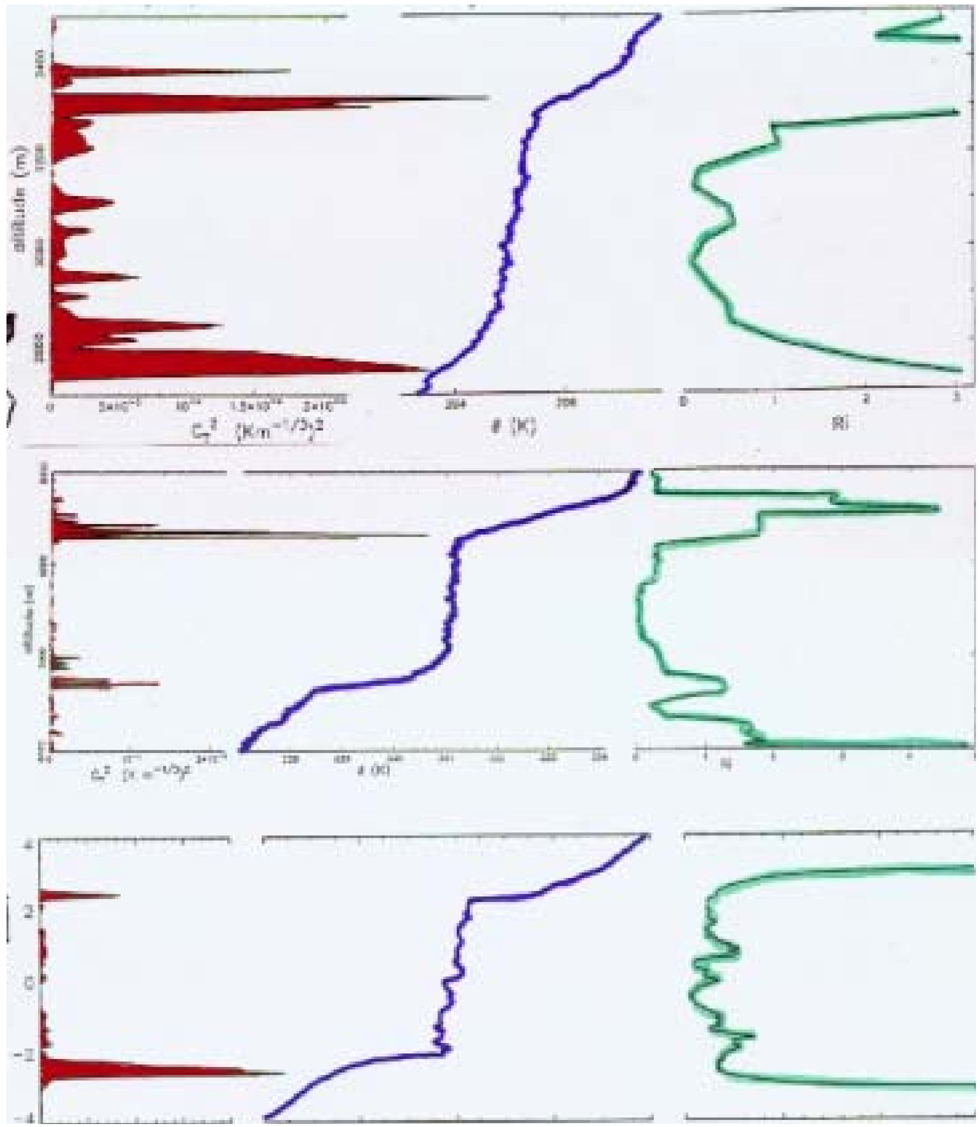
Chen, Kelley, Gibson-Wilde, Werne & Beland, *Annales Geophysicae*, 2001



Wind shear: Balloon Comparison

Coulman, Vernin & Fuchs, *Applied Optics* 34 5461 (1995)

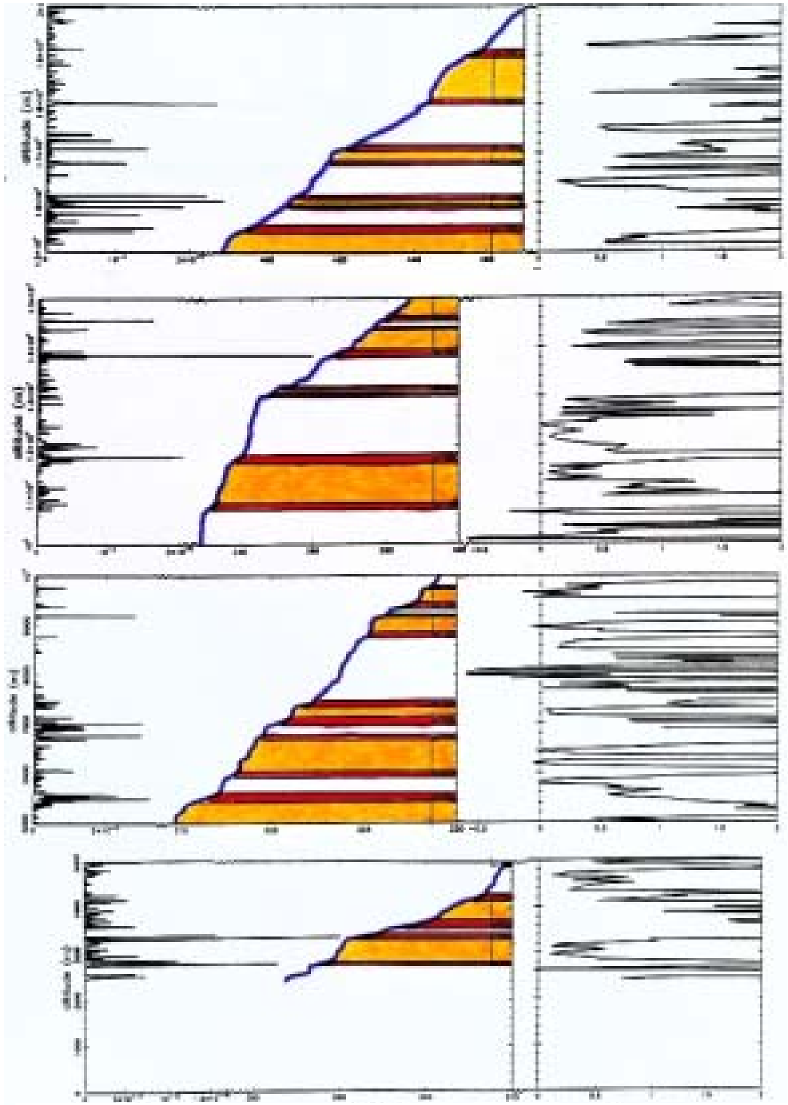




C_T^2 T R_i

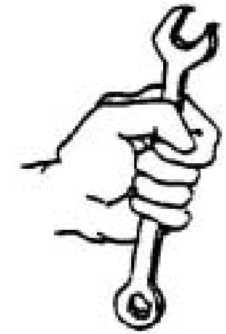
Mixing Layers through the Troposphere and Stratosphere

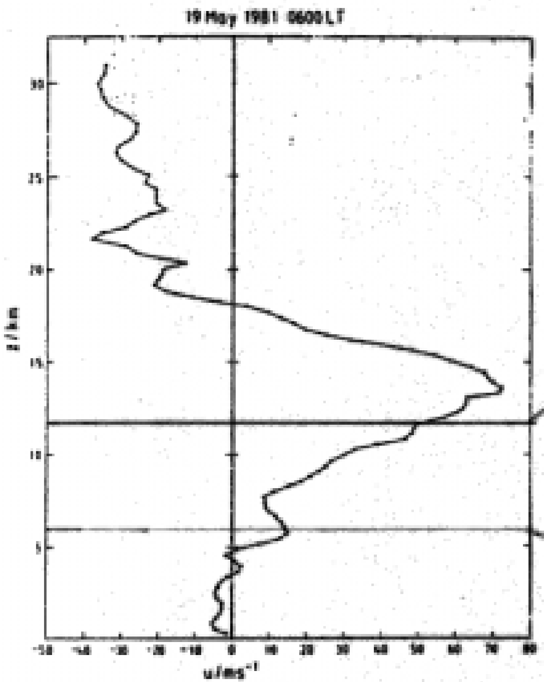
Coulman, Vernin & Fuchs, *Applied Optics* 34 5461 (1995)



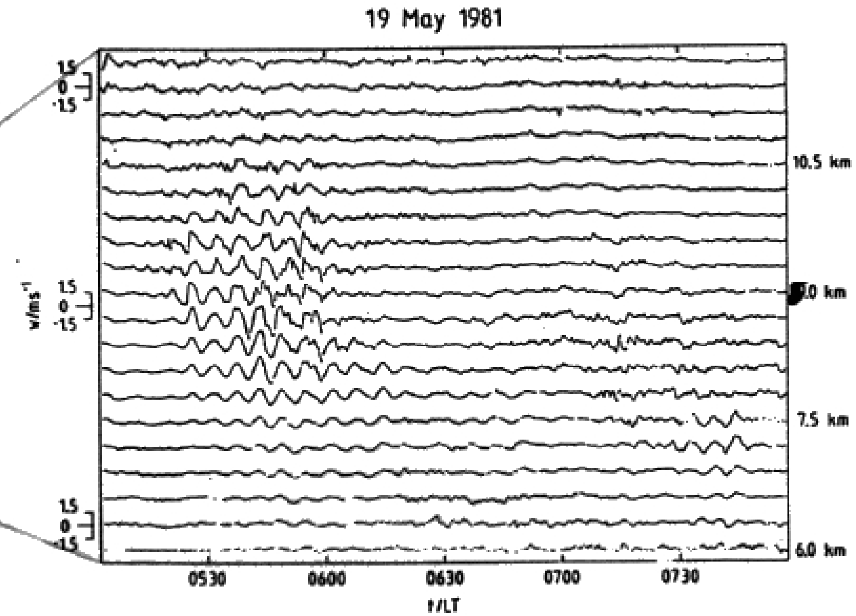
C_T^2 T R_i

Sousy Radar over Harz, Germany





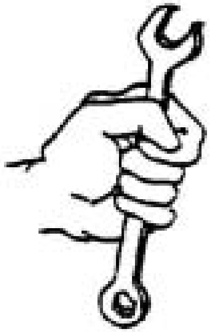
Height profile of west wind (positive towards east) deduced fro radiosonde data taken at San Juan (Puerto Rico). Height values always refer to me: sea level.

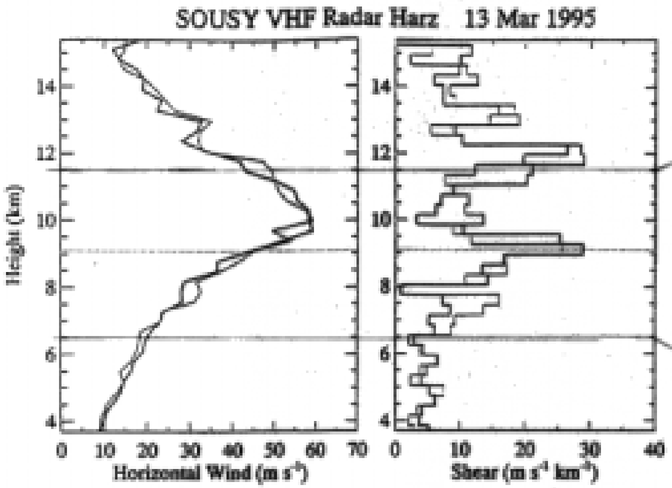


Time series of vertical velocity w at 20 consecutive heights observed with the SOUSY VHF radar at Arecibo (Puerto Rico).

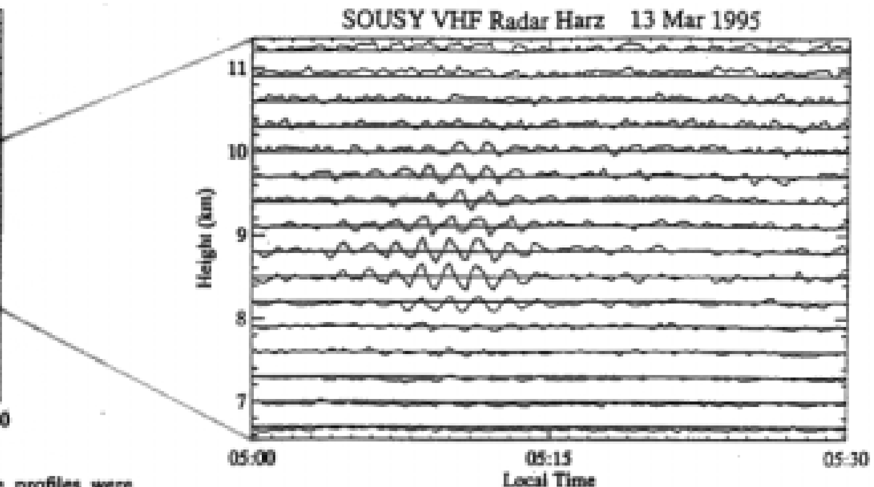
Chilson, Muschinski & Schmidt, *Radio Science*, 32, 1997

Sousy Radar at Arecibo





Height profiles of the horizontal wind and the vertical wind shear. The profiles were measured at 0427 LT (bold lines) and 0534 LT (light lines). The horizontal dashed lines mark the 9.1-km height.



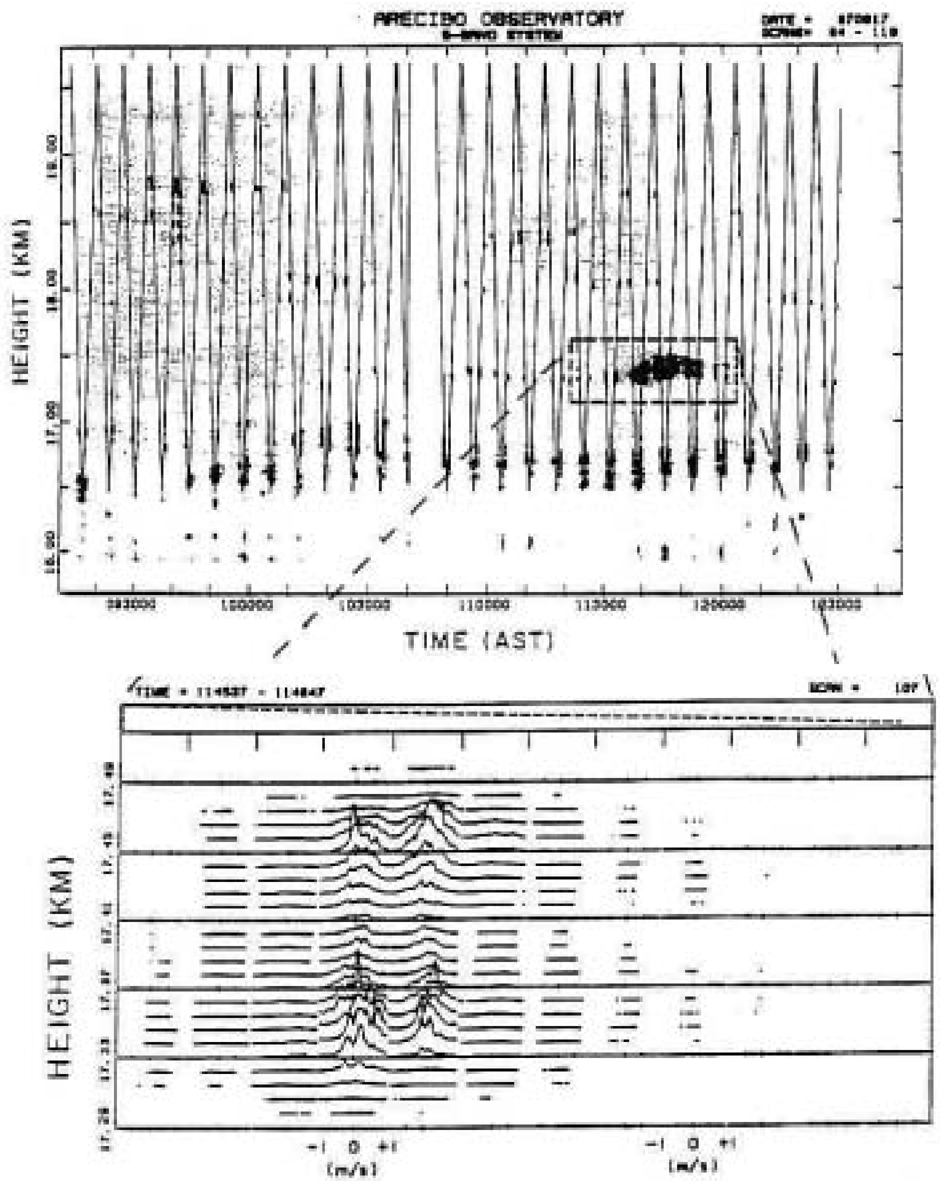
Unfiltered radial velocities measured while the radar beam was oriented vertically. The large-amplitude oscillations mark the occurrence of a Kelvin-Helmholtz instability (KHI). The velocities have been scaled such that 1 m s^{-1} corresponds to 100 m.

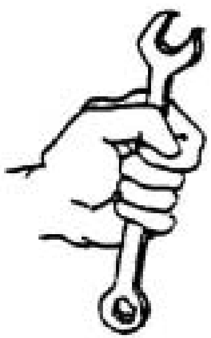
Rüster and Klostermeyer, *Geophys. Astrophys. Fluid Dynamics*, 26, 1983

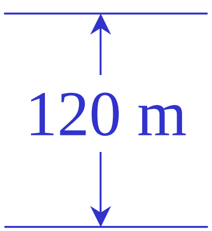
Radar Backscatter

Ierkic, Woodman & Perillat, *Radio Science* 25, 941 (1990)

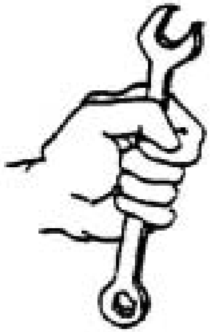
Re ~ 10^6-10^7



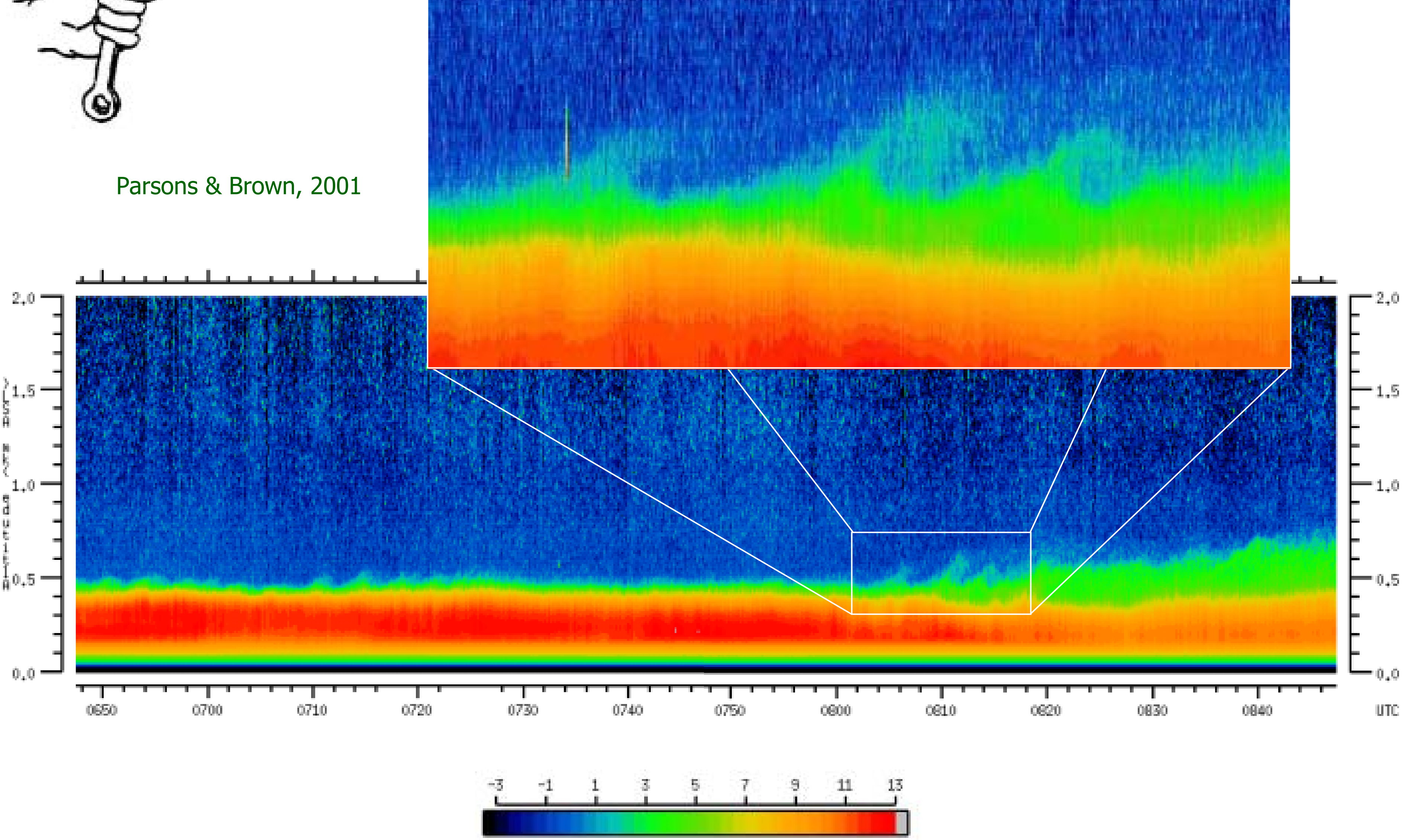




Kelvin-Helmholtz at VTMX

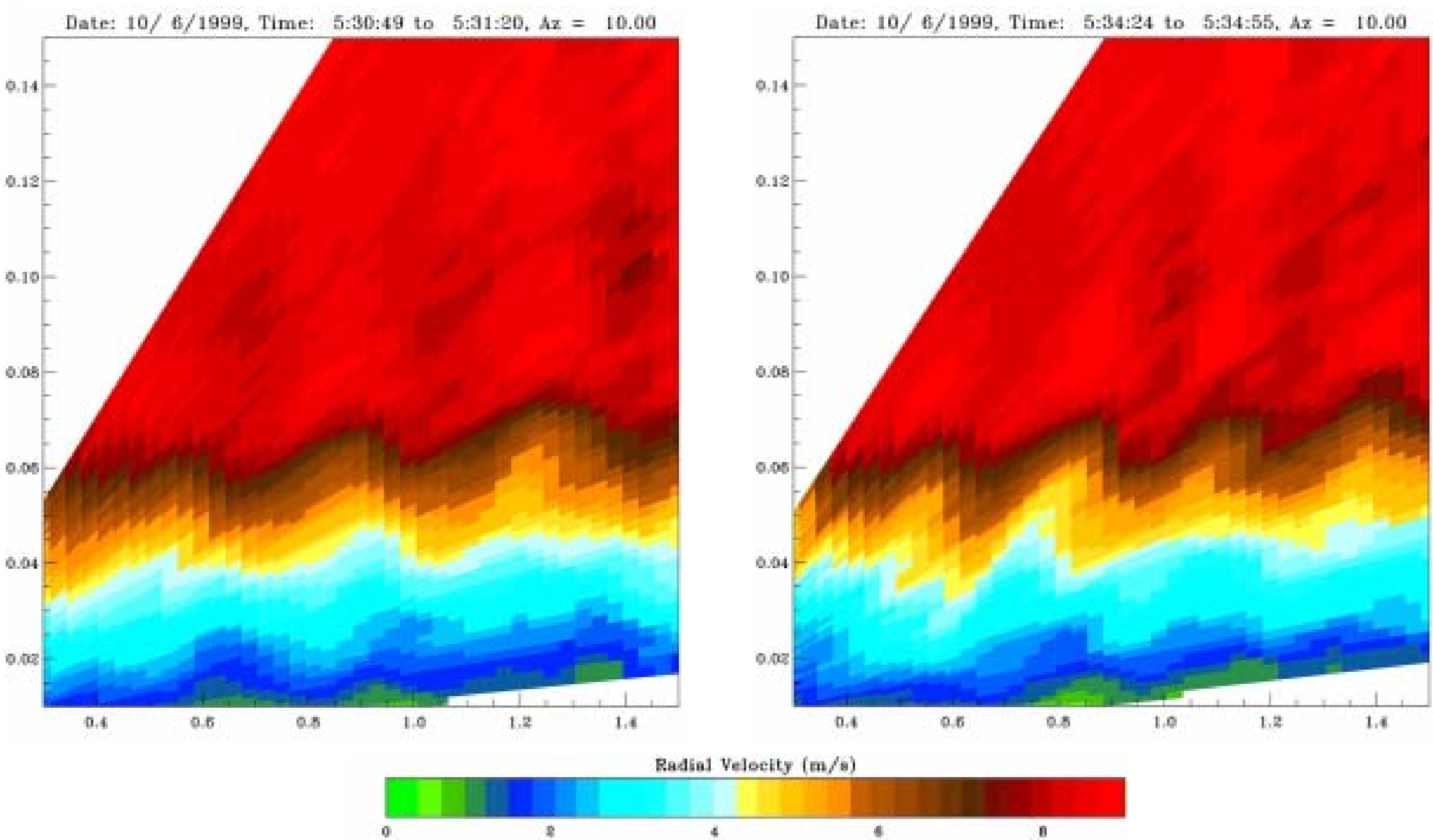


Parsons & Brown, 2001



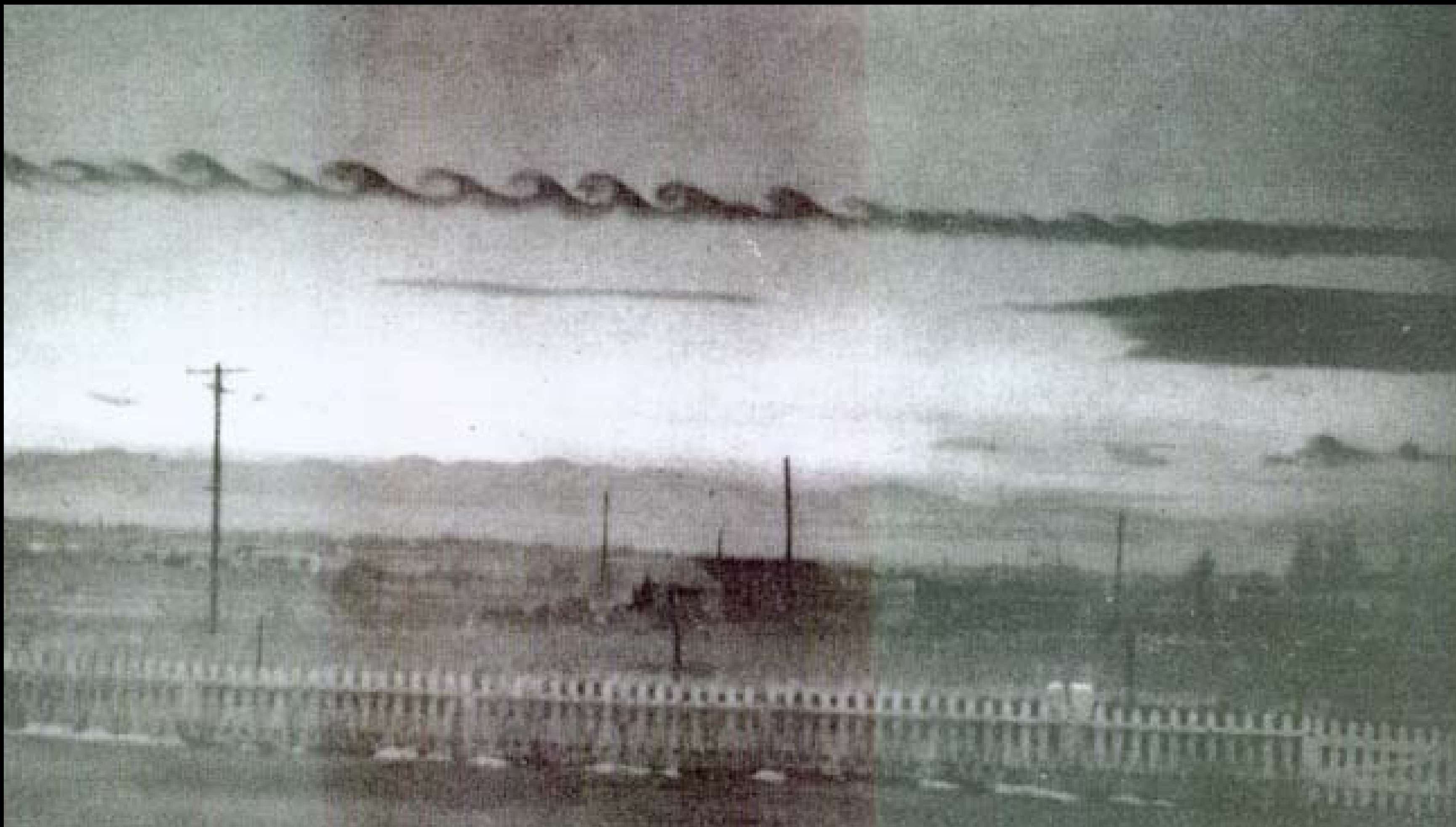
Kelvin-Helmholtz at CASES-99

Blumen, Banta, Burns, Fritts, Newsom, Poulos, Sun, *Dyn. Atmos. Oceans*, 2001



Estes Park, Colorado, 1979 (photo by Bob Perney)

Colorado Springs, Colorado, 2000 (photo by Tye Parzybok)



Denver, Colorado, 1953 (photo by Paul E. Branstine)

Noctilucent Clouds, Kustavi, Finland, 1989 (photo by Pekka Parviainen)

DNS Efforts

- Validate simulations
- Characterize/quantify atmospheric turbulence

Phase Screen Specification

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- Quantify non-Kolmogorov effects

Turbulence Simulation Algorithm Development

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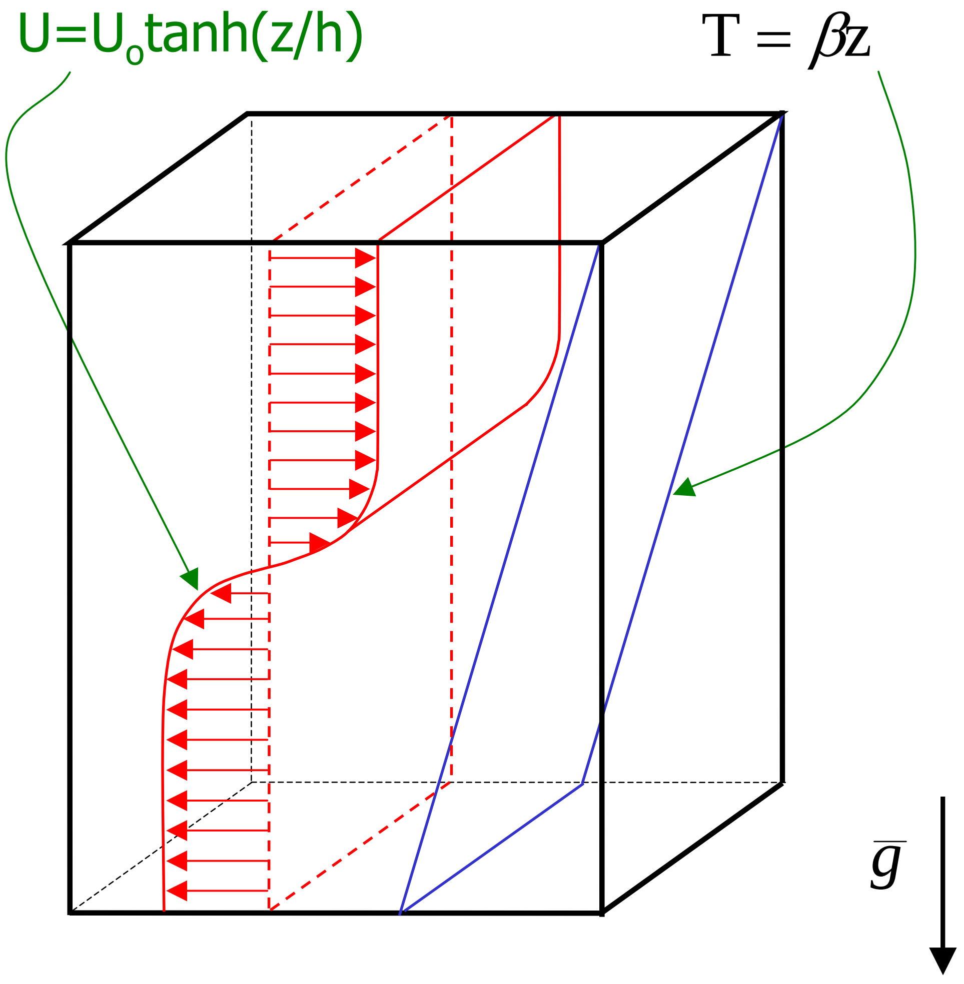
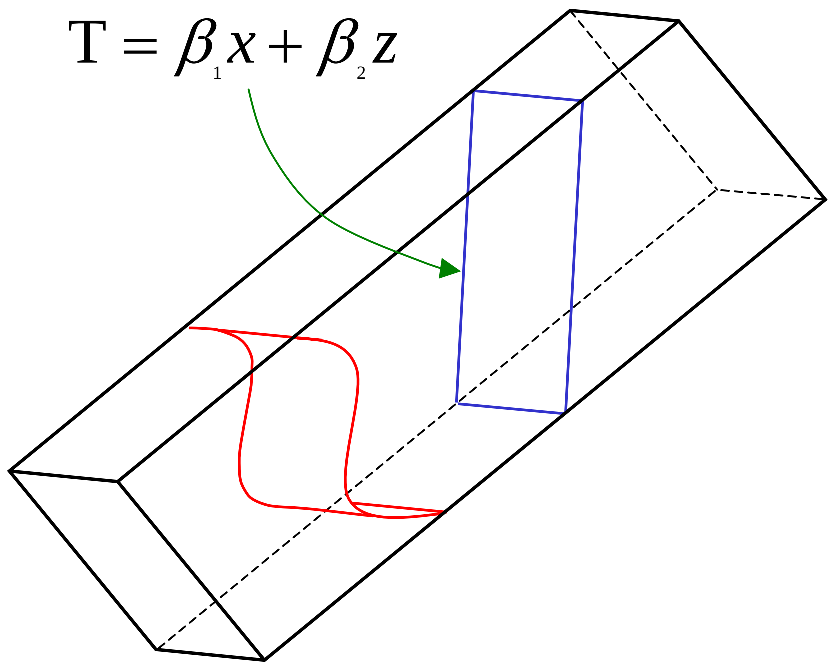
Wind Shear



Gravity-Wave Breaking



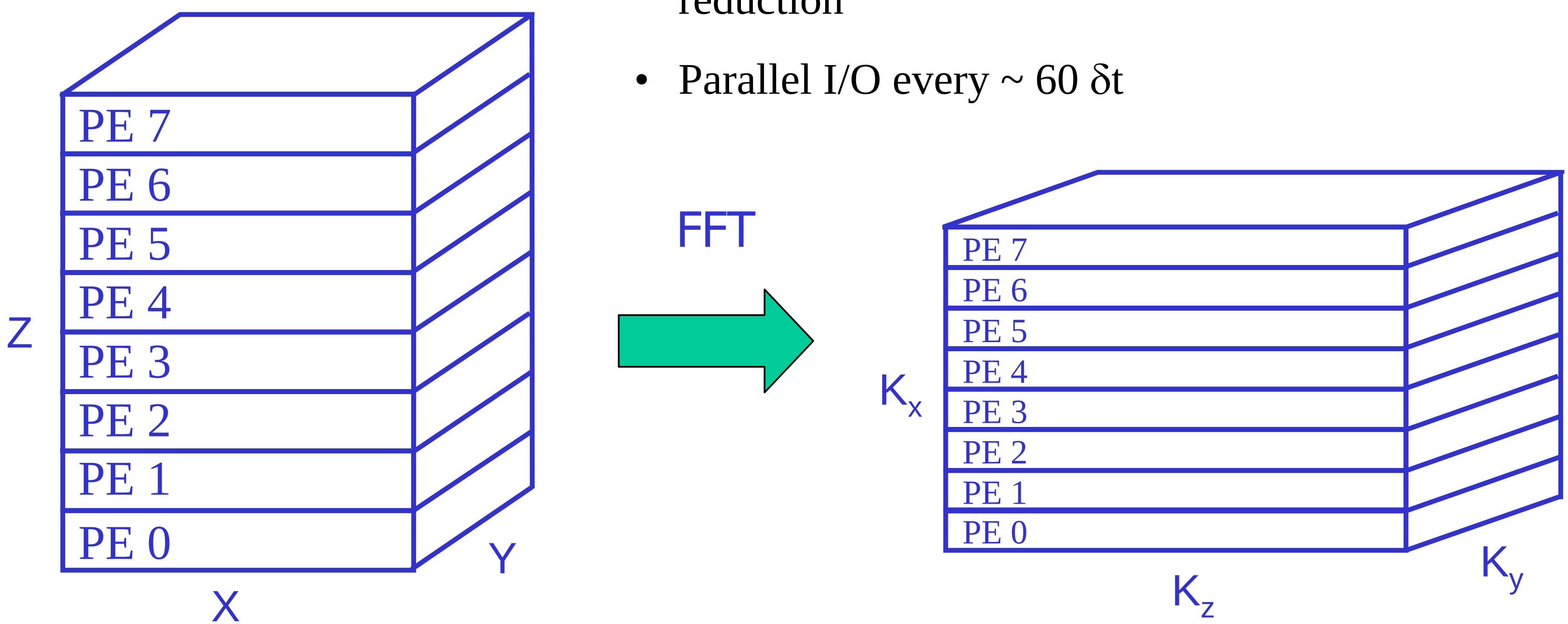
3D Incompressible Navier-Stokes Solver

$$\partial_t \mathrm{u} + \omega \times \mathrm{u} = \mathrm{Re}^{-1}\Delta \mathrm{u} - \nabla \mathrm{P} + \mathrm{R_i}\,\Theta$$

$$\partial_t \Theta + \mathrm{u} \cdot \nabla \Theta = \mathrm{Pe}^{-1}\Delta\Theta$$

$$\nabla \cdot \mathrm{u} = 0$$

- Stream-function/vorticity formulation
- Fully spectral (3D FFT's = 75% computation)
- Radix 2,3,4,5 FFT's
- Spectral modes and NCPUs must be commensurate
- Communication: shmem, global transpose, data reduction
- Parallel I/O every ~ 60 δt



Headaches, woes, and what to do about them.

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- Typical 20-hour run on 500 processors generates 800 Gigabytes of data and over 70,000 individual files.
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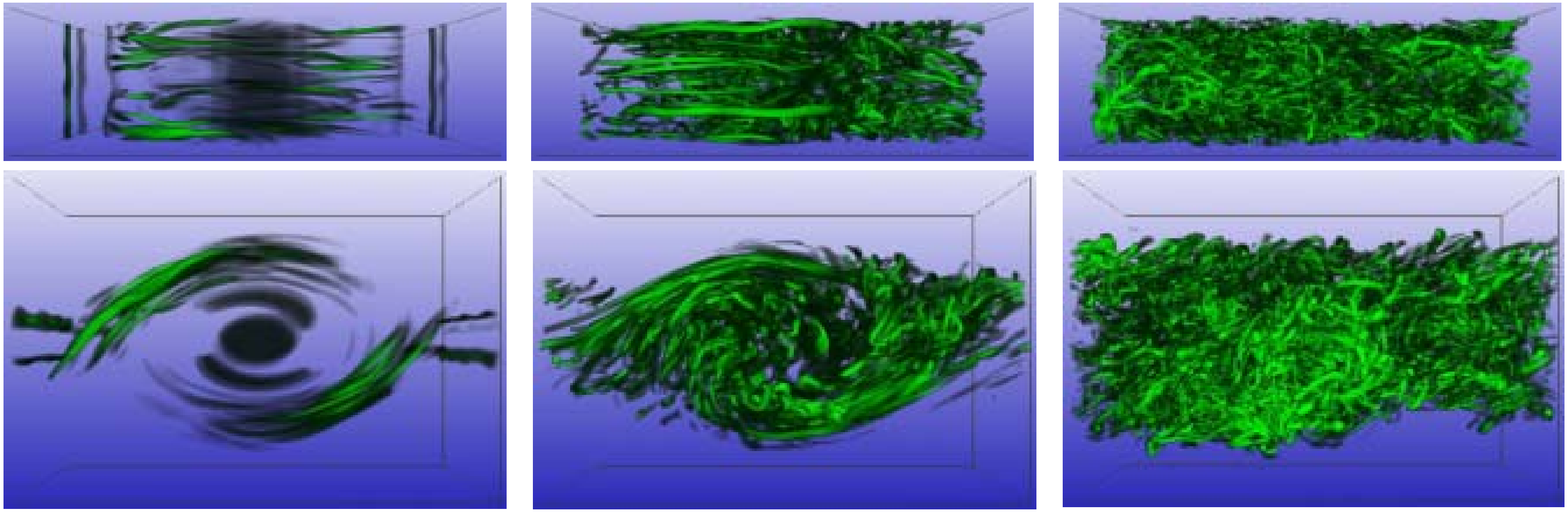
- Interactive performance can be poor during large production runs.
- Center non-uniformity contributes to drudgery.
- Elaborate scripts automate job specification, source-code editing, compilation, and submission as well runtime data transfers and migration off-line to archival storage.
- FORTRAN code and accompanying Perl scripts run without modification at 6 supercomputer centers and 4 MPP architectures (T3E, O3k, SP, Compaq).
- DoD has adopted our batch-preparation and archival-storage routines as a standard (PST, Werne, Gourlay).

Vortex-Tube Morphology

Werne, Meyer, Bizon & Fritts, 2001

Wind shear



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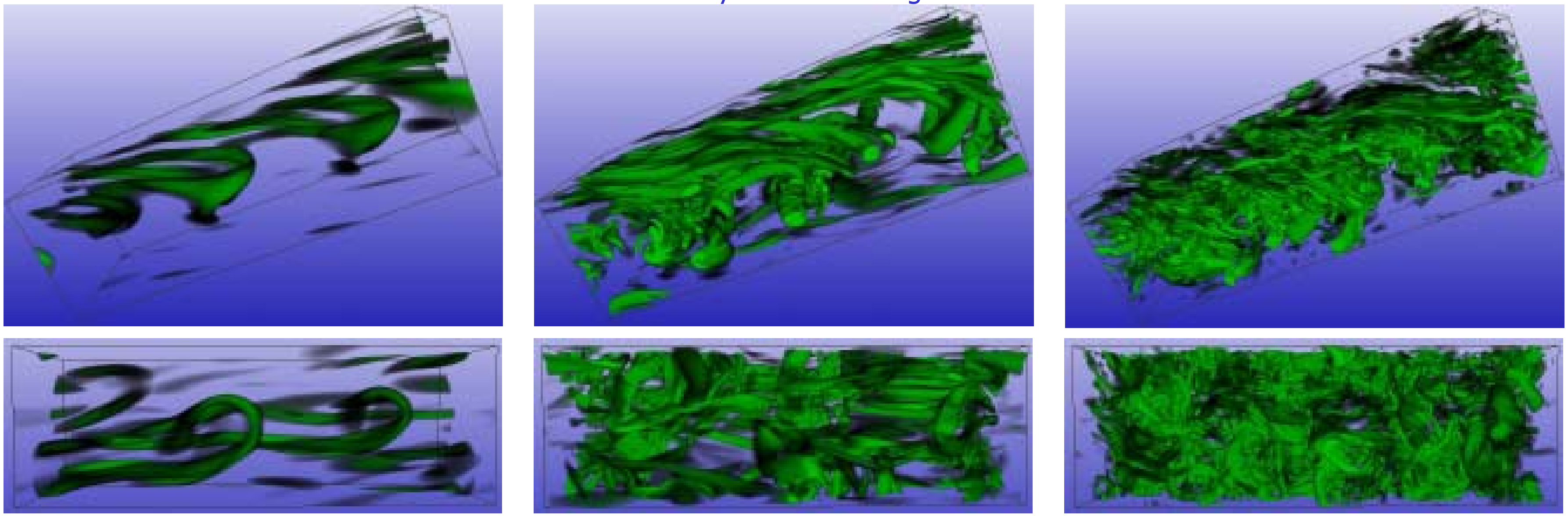
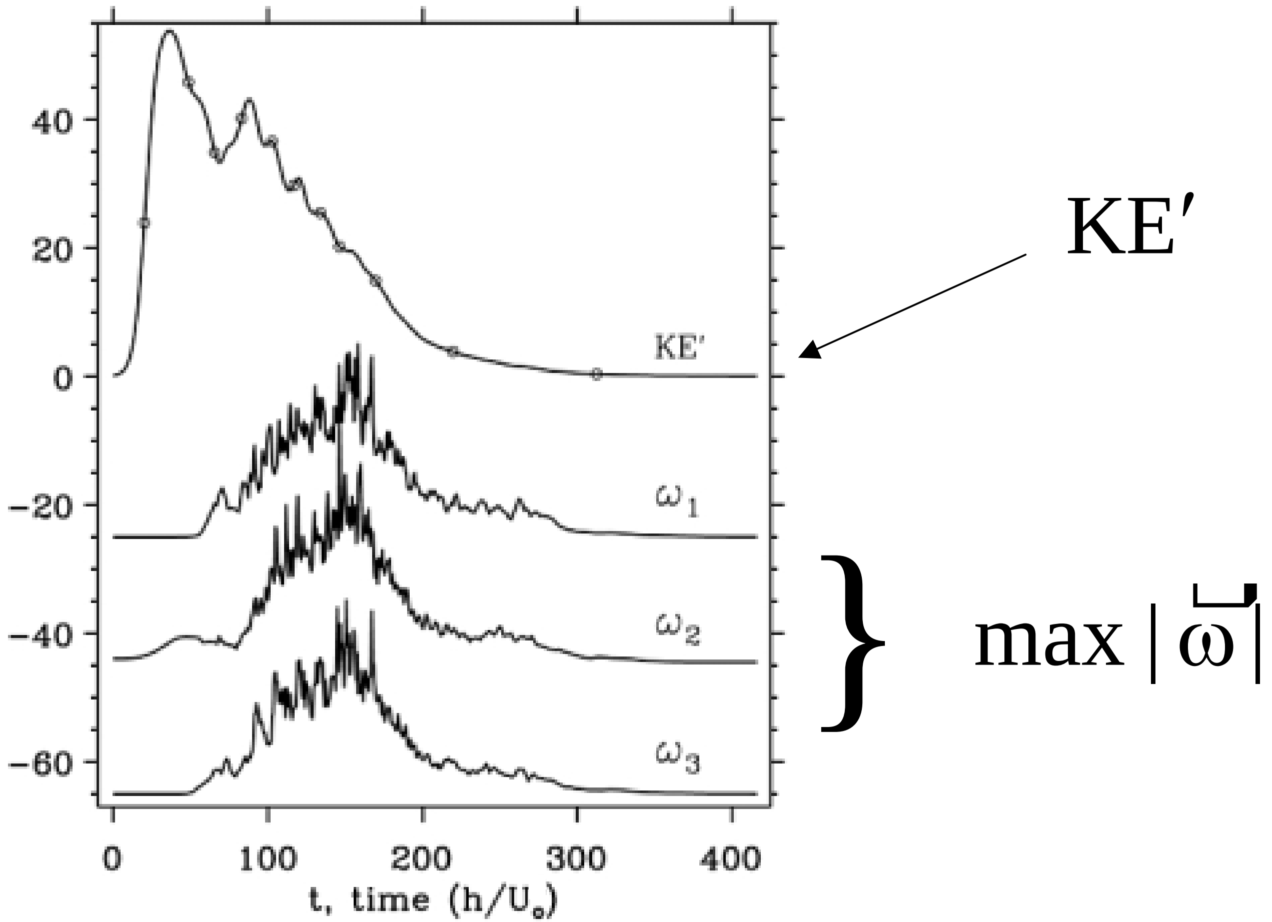


Kelvin-Helmholtz: Evolution



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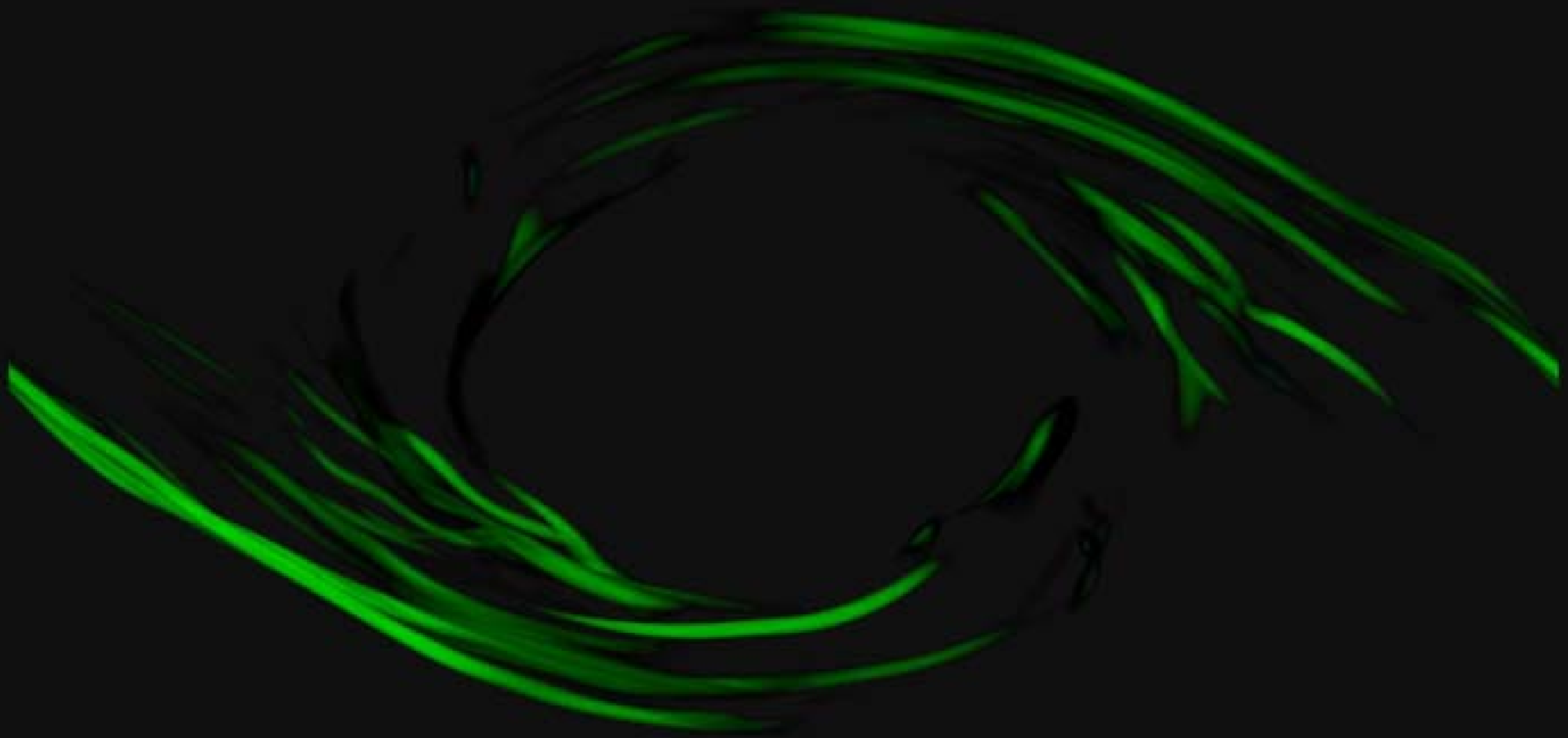

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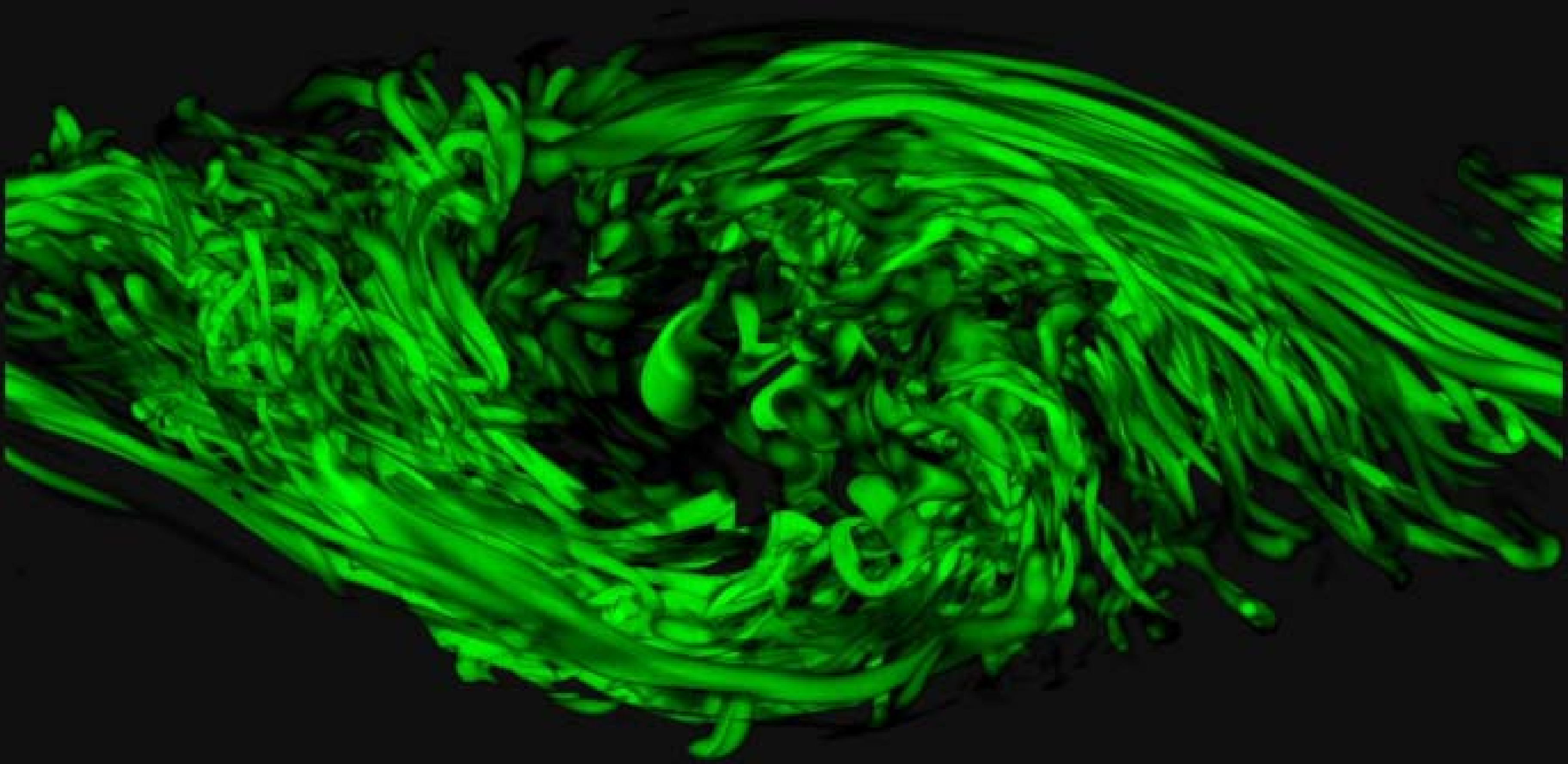

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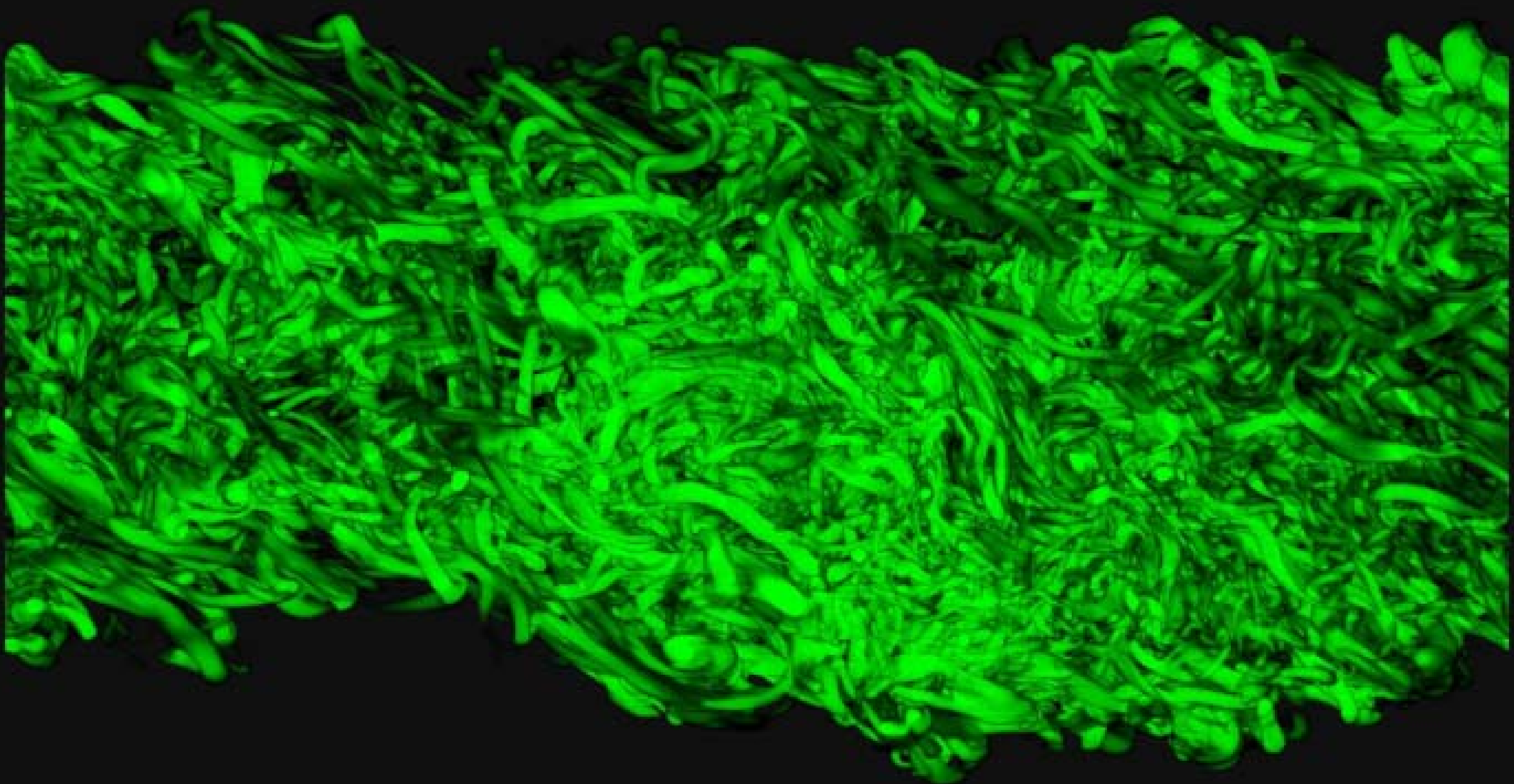

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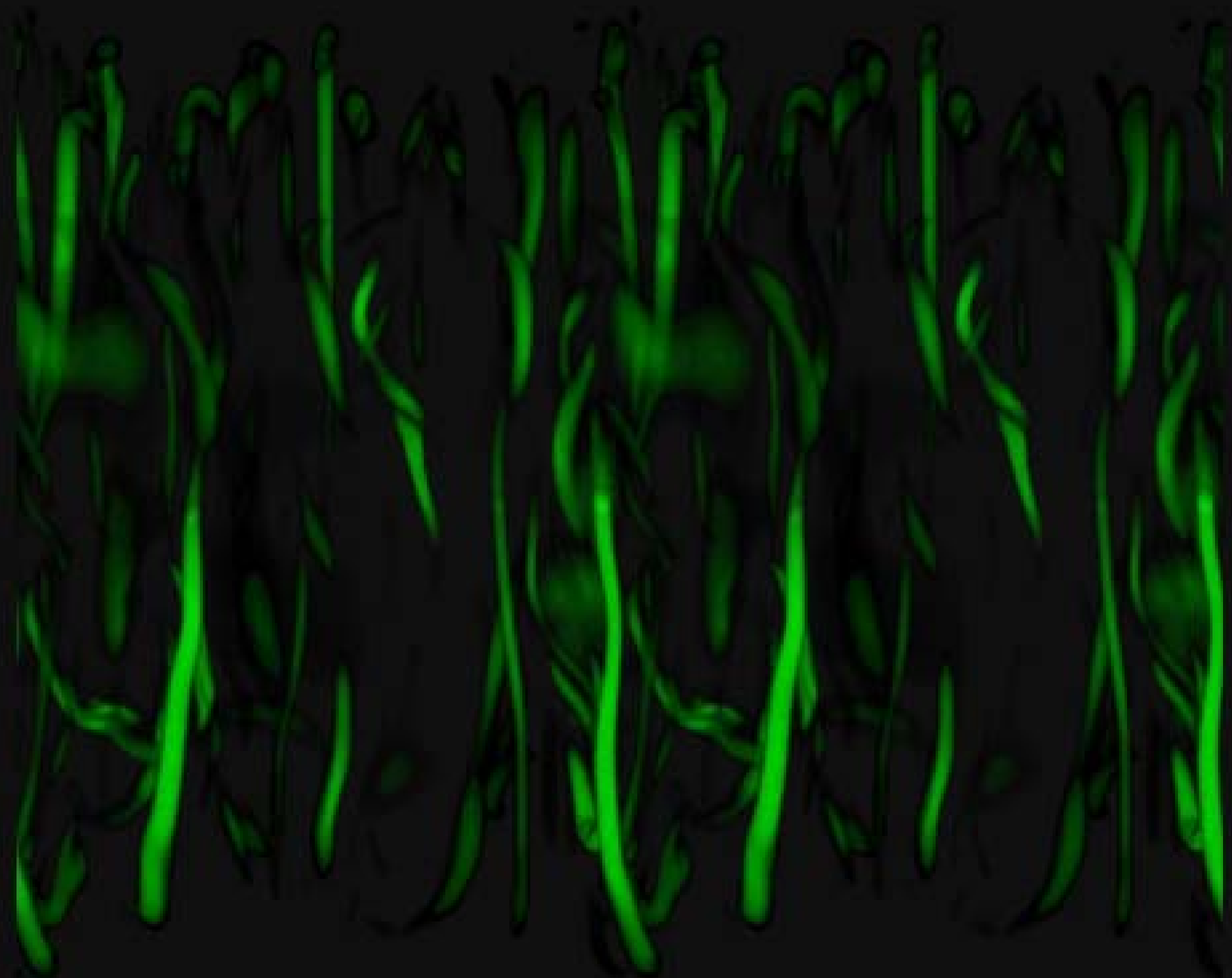

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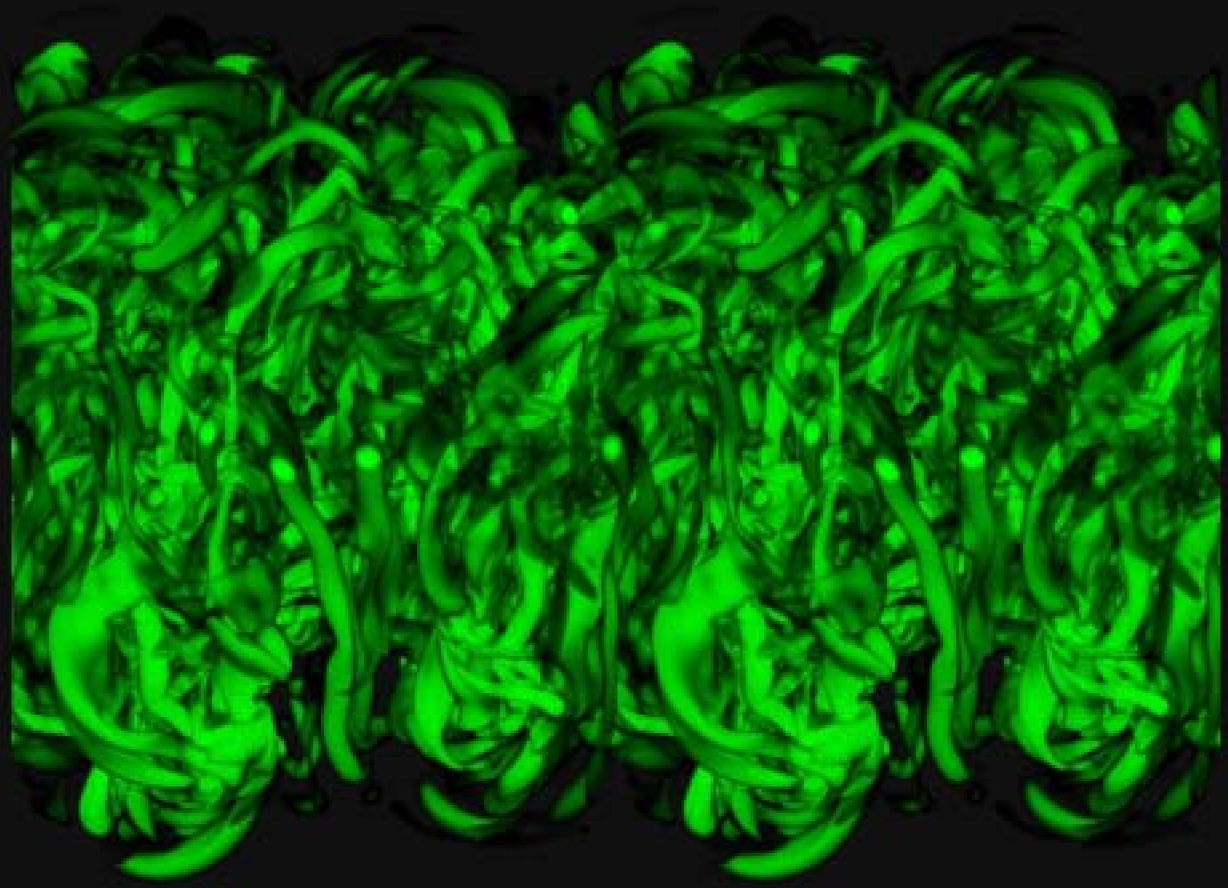

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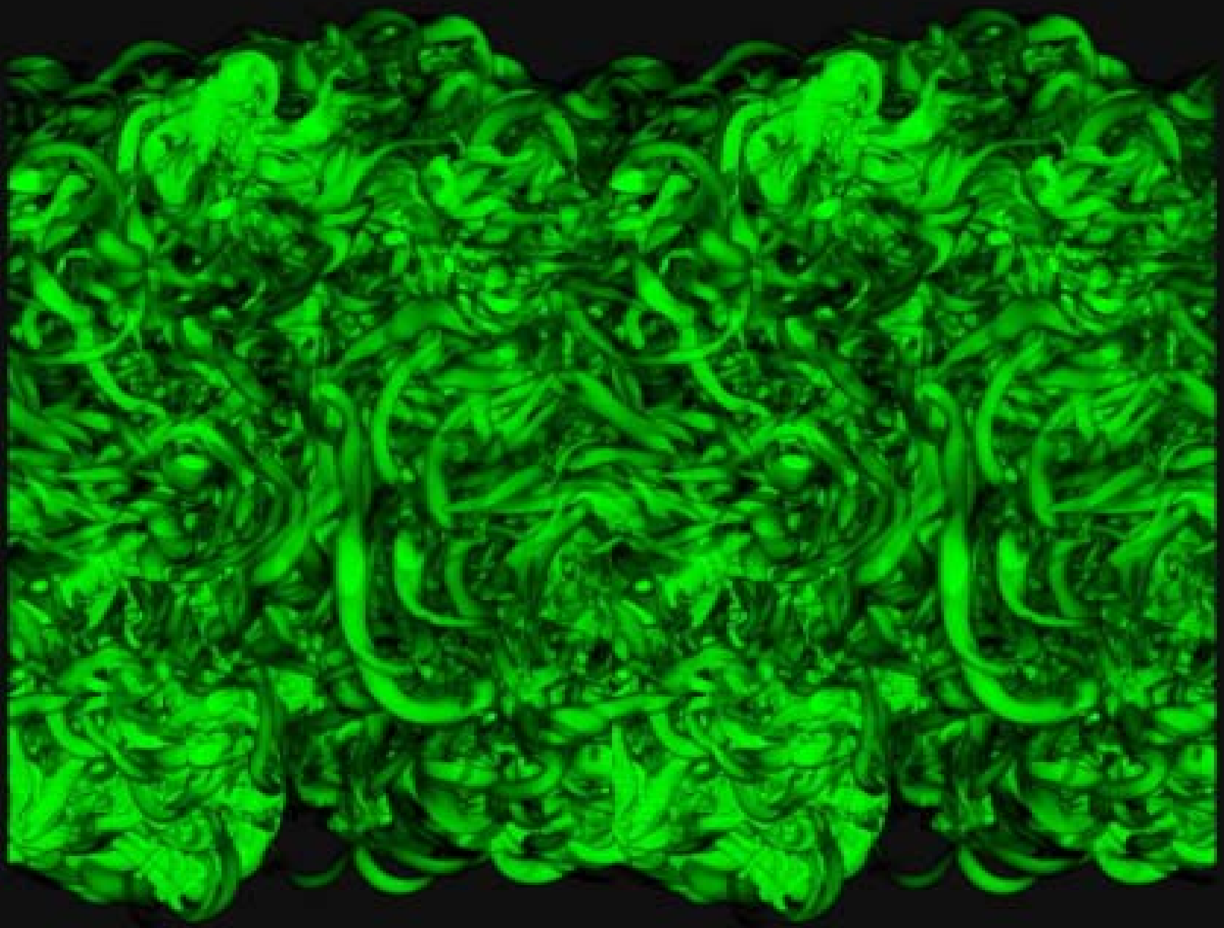

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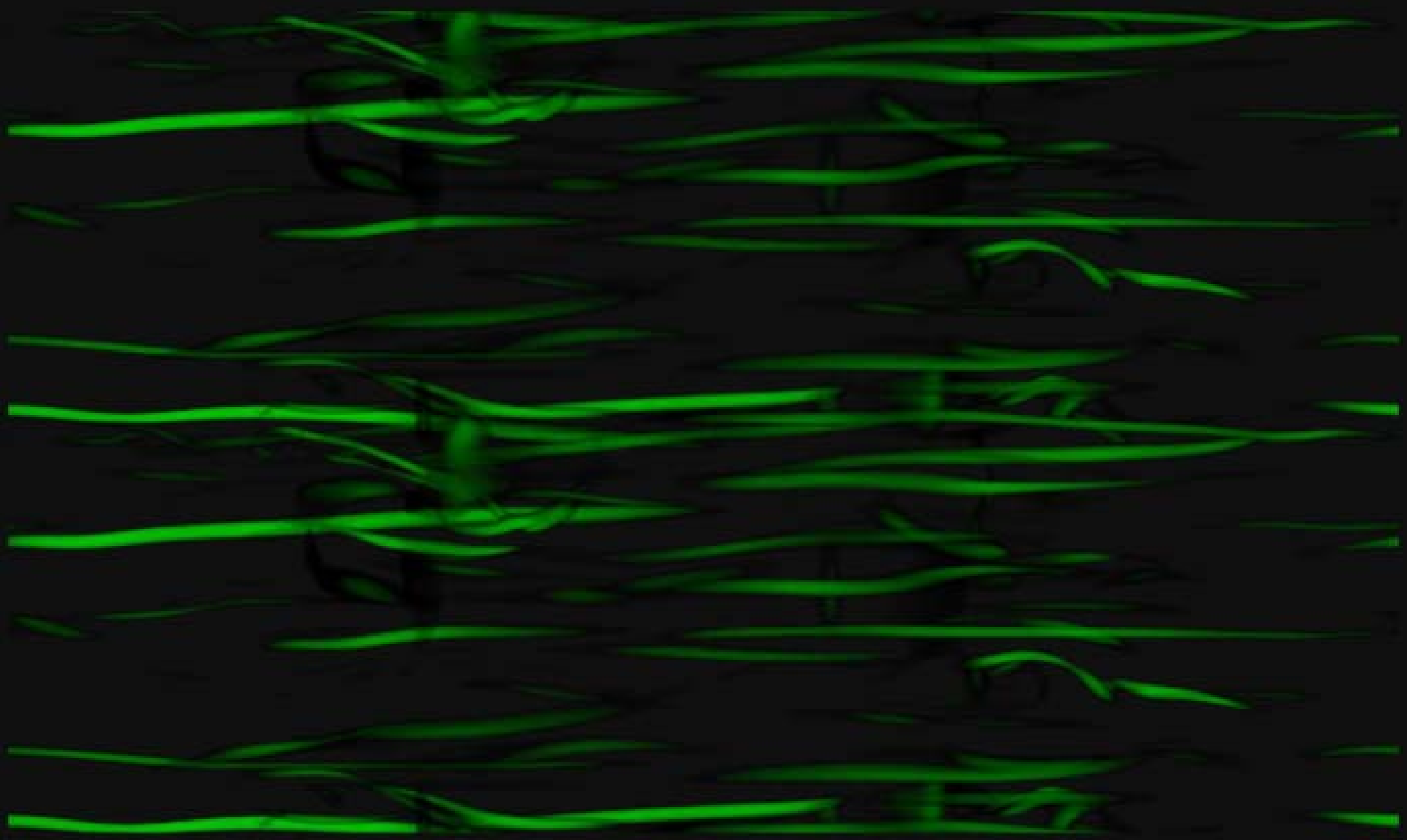

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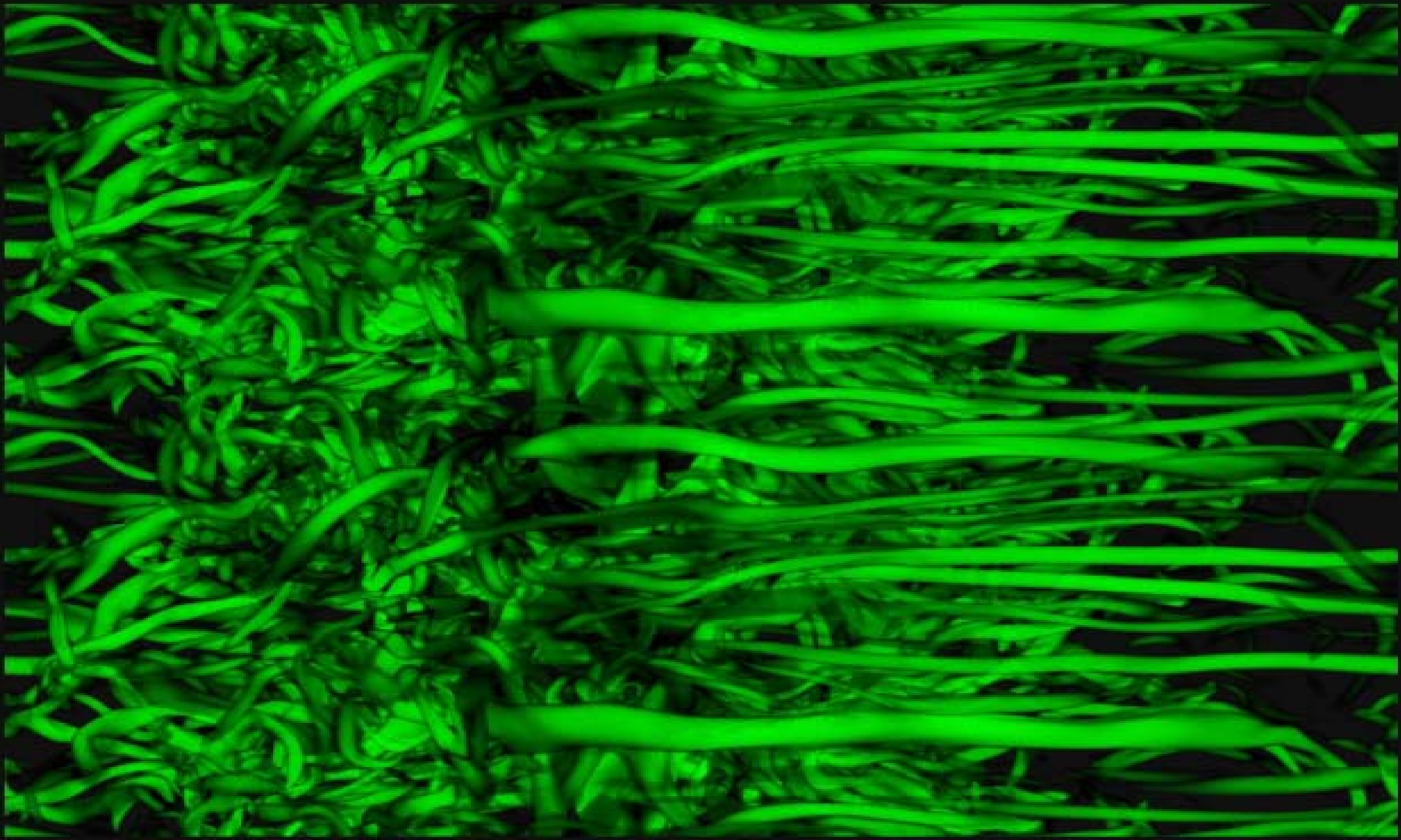

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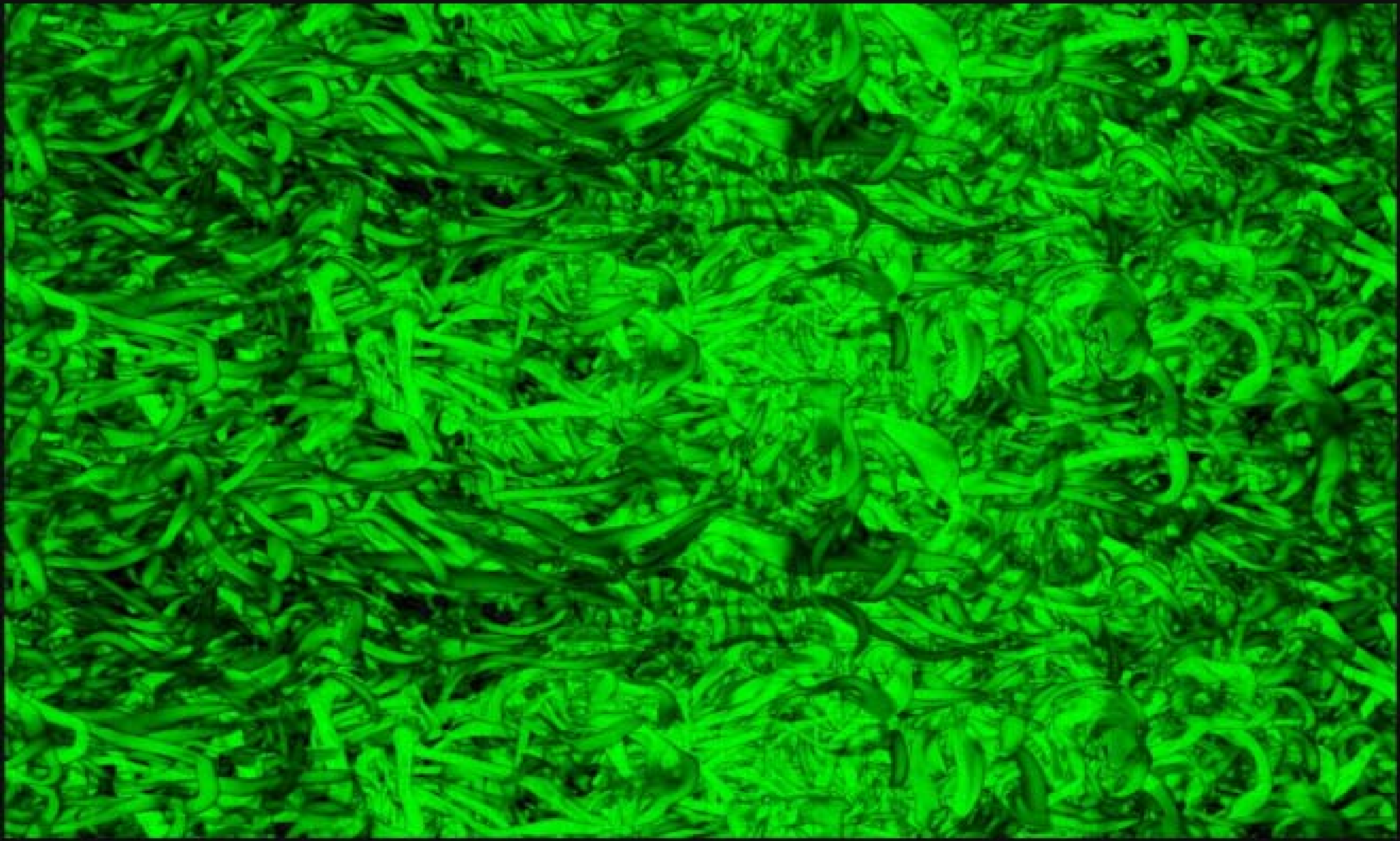
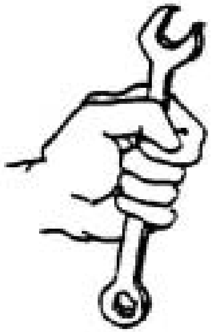


Wind shear: Balloon Comparison



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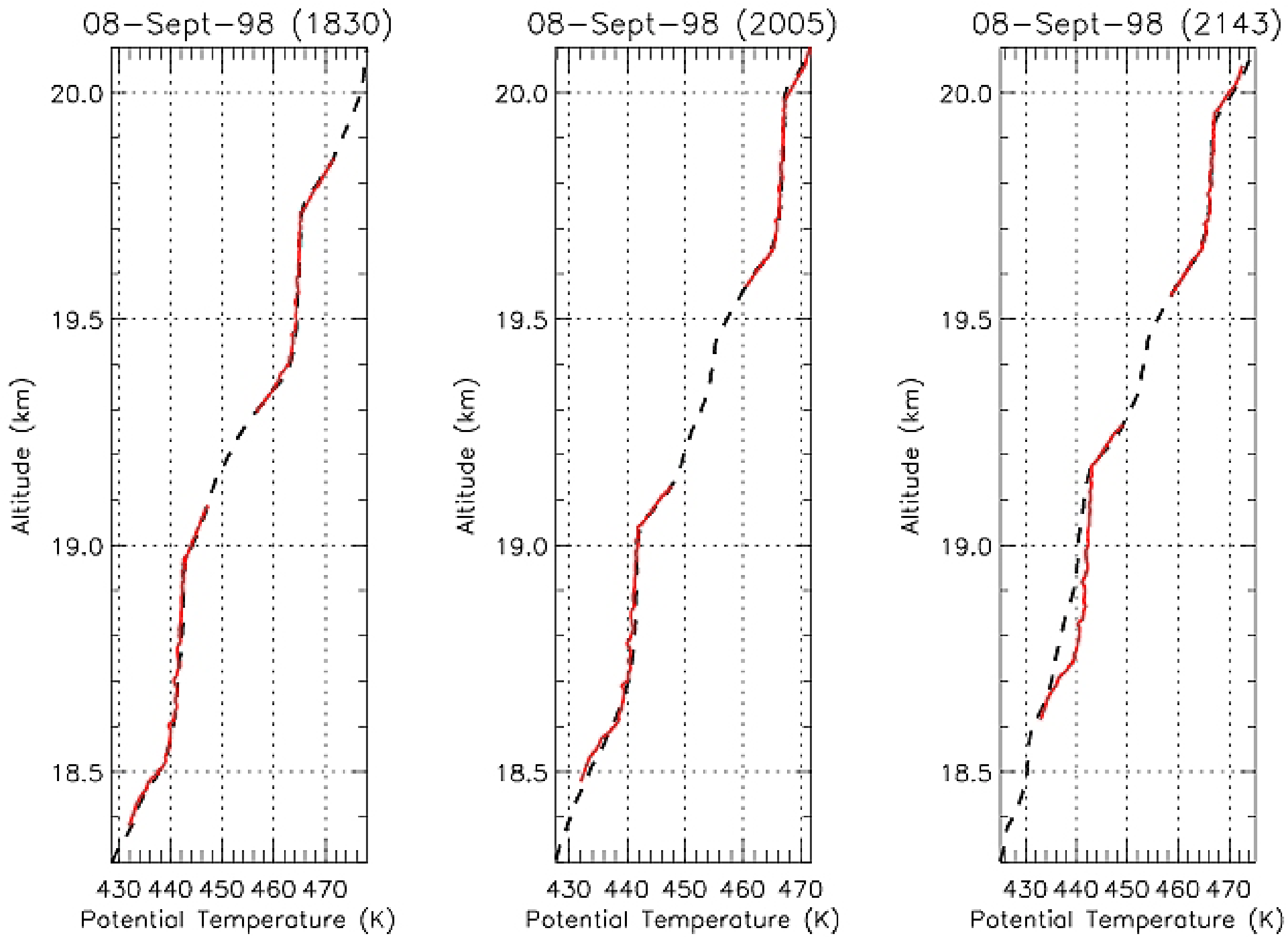


Wind Shear: flow decomposition

$$T = \overline{T} + \tilde{T} + T'$$

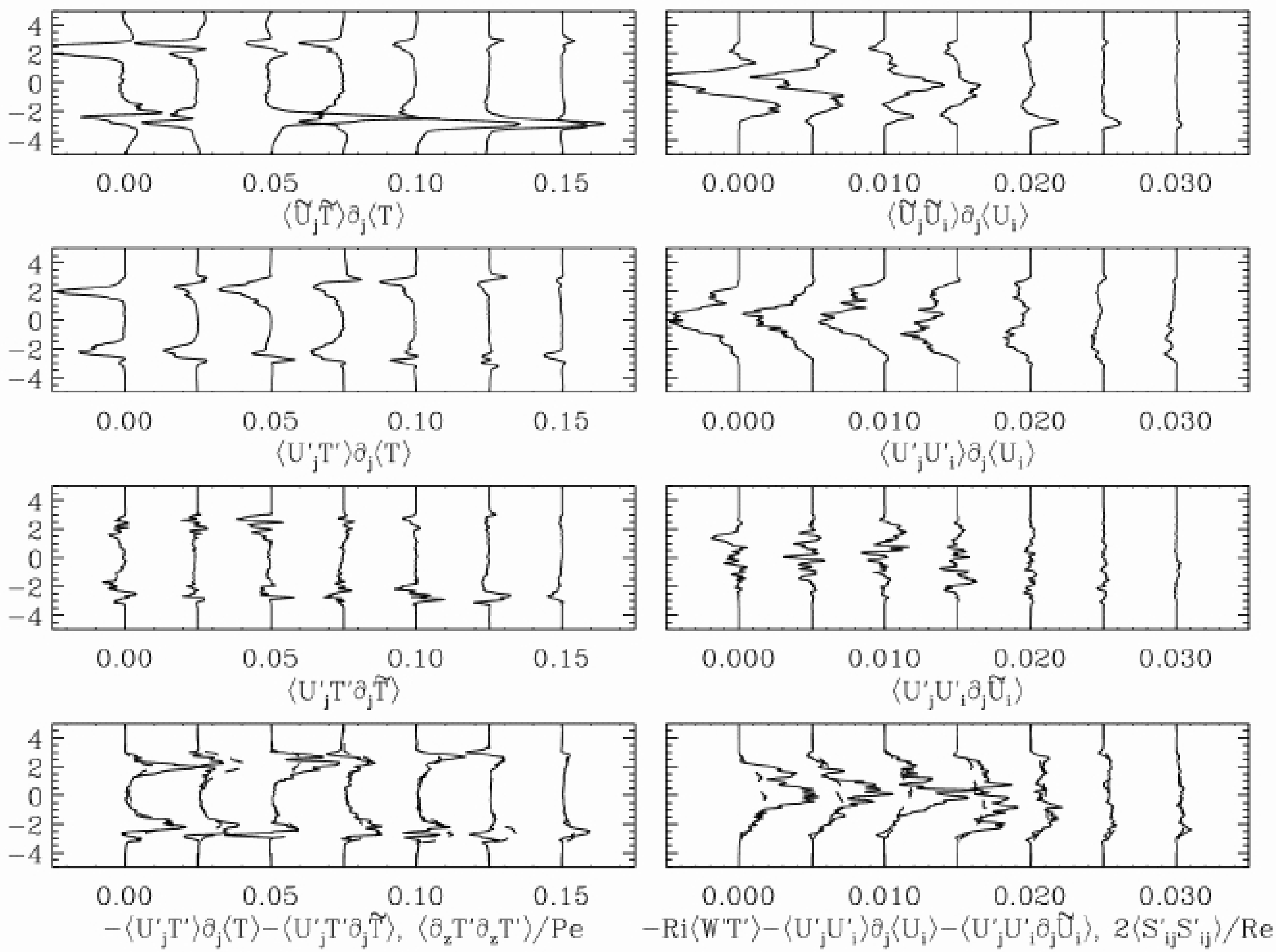
3D flow field

fluctuations

horizontal average T

spanwise average of $T-\overline{T}$

Wind shear: production and dissipation



Stratified Turbulence Theory: thermal and viscous dissipation

Kolmogorov 1941

$$\Delta_r T^2 = C_\theta \, \varepsilon^{-1/3} \, \chi \, r^{2/3}$$

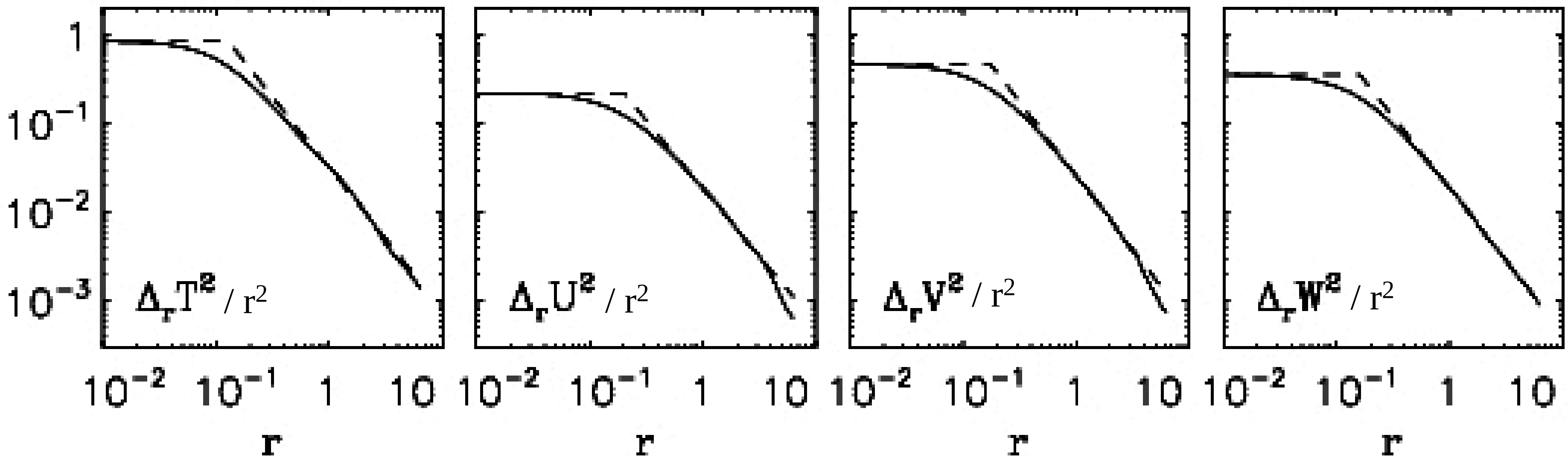
$$\Delta_r U^2 = C \, \varepsilon^{2/3} \, r^{2/3}$$

Bolgiano 1959

$$\Delta_r T^2 = C_\theta \, Ri^{-2/5} \, \chi^{4/5} \, r^{2/5}$$

$$\Delta_r U^2 = C \, Ri^{4/5} \, \chi^{2/5} \, r^{6/5}$$

Wind shear: 2nd-order structure-function fits



Wind shear: 2nd-order structure-function fits

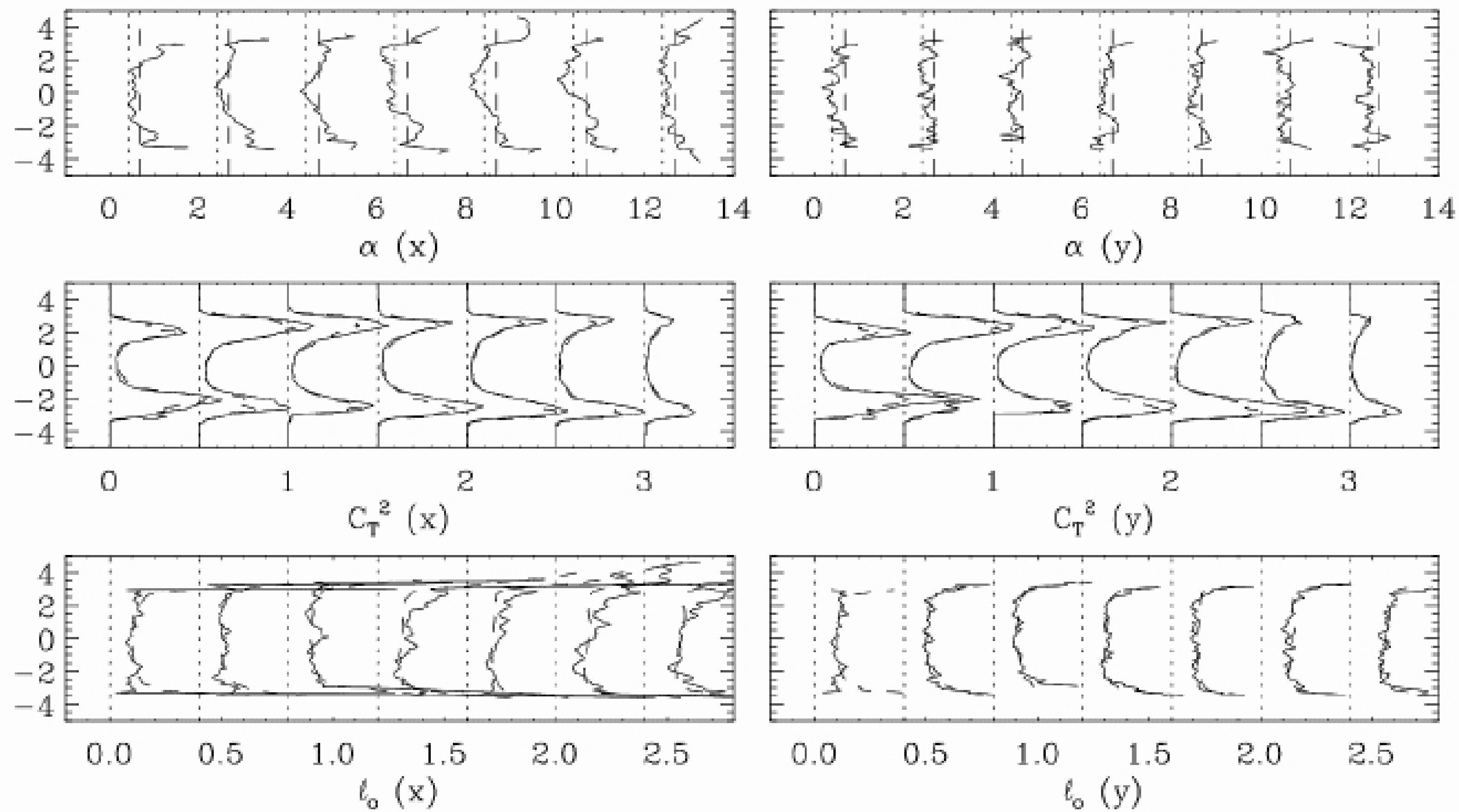
$$\Delta_r T^2 = C\, r^\alpha$$

Werne & Fritts, 2000

α

C

●$_o$



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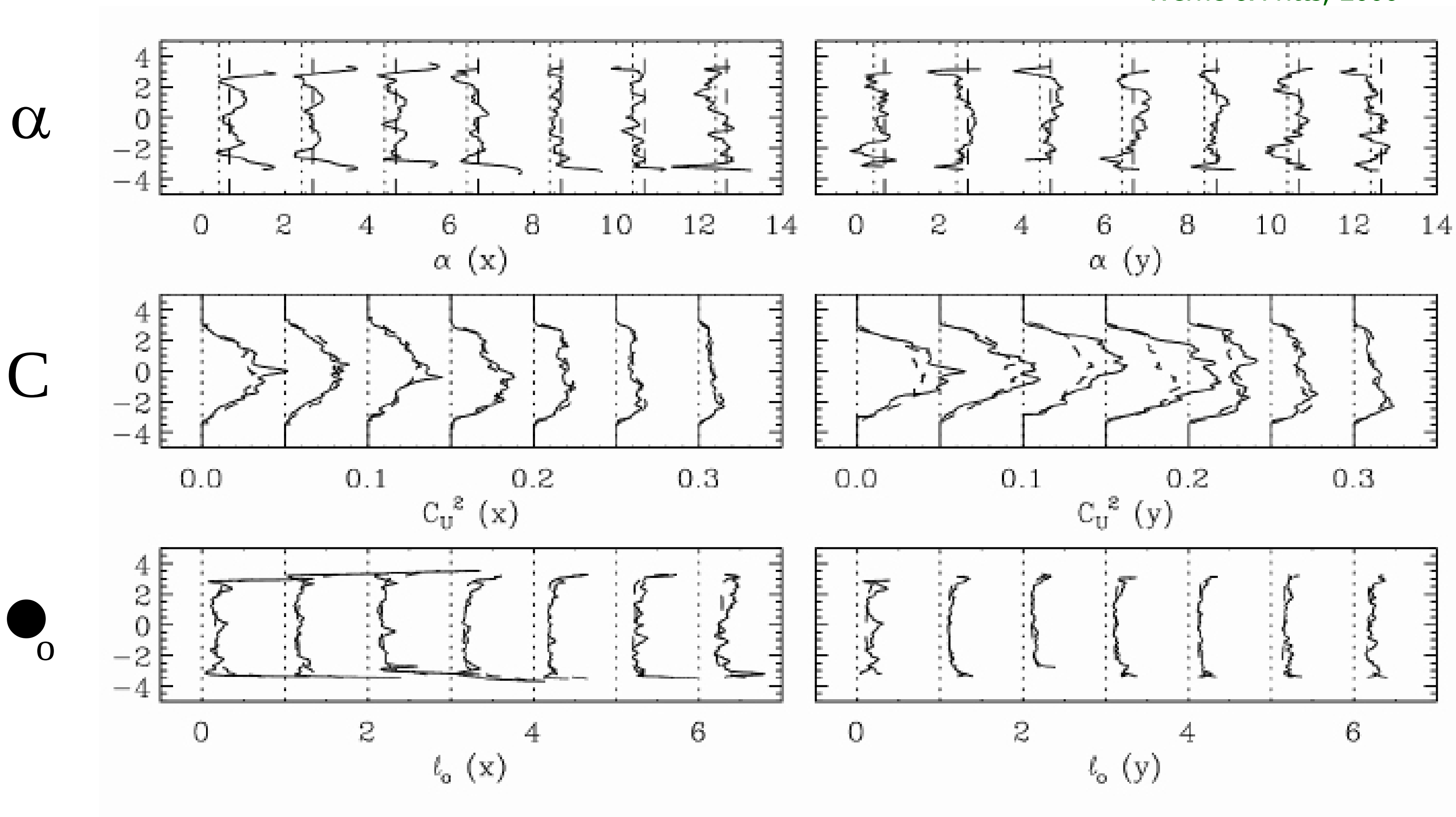
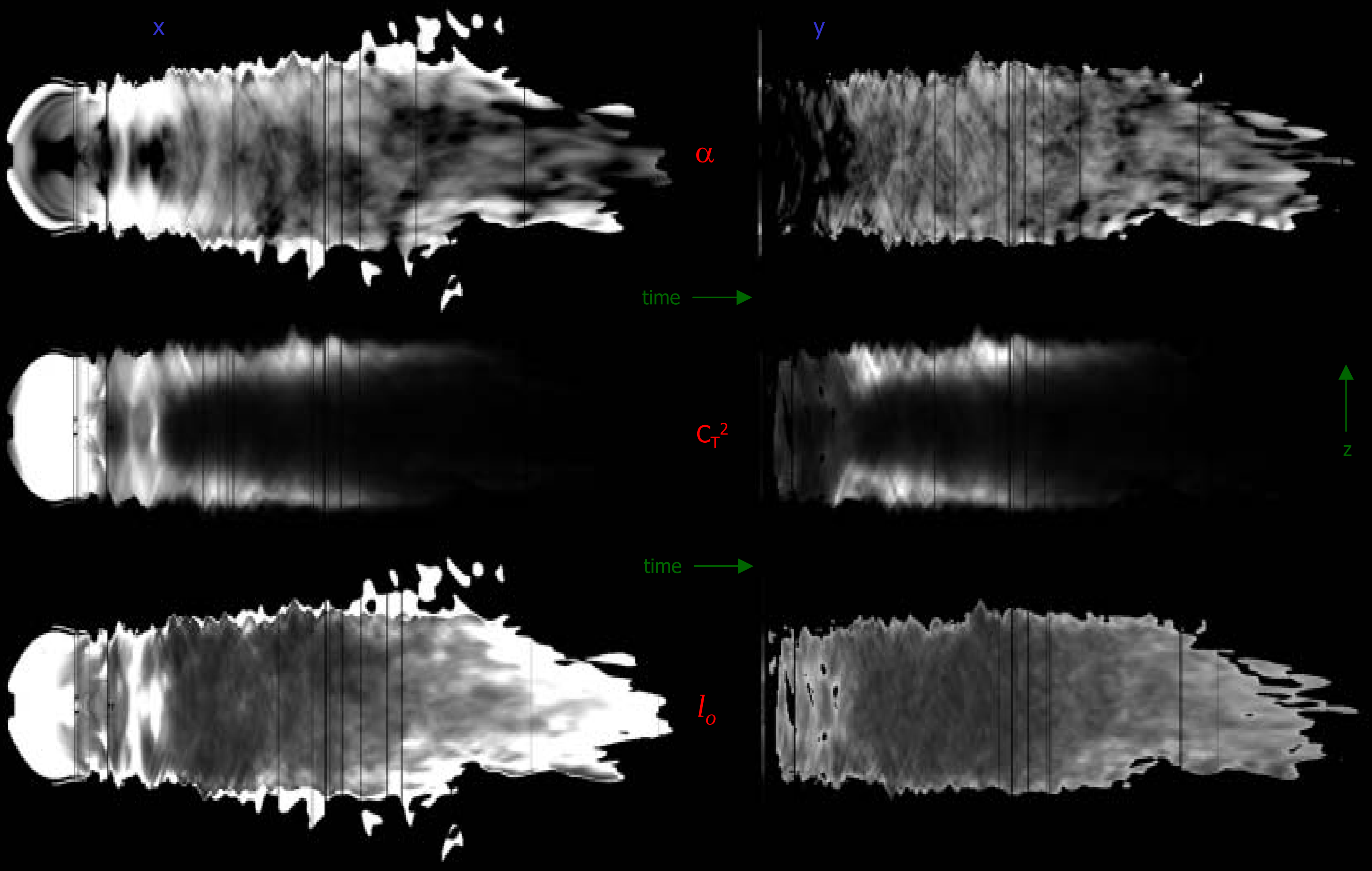
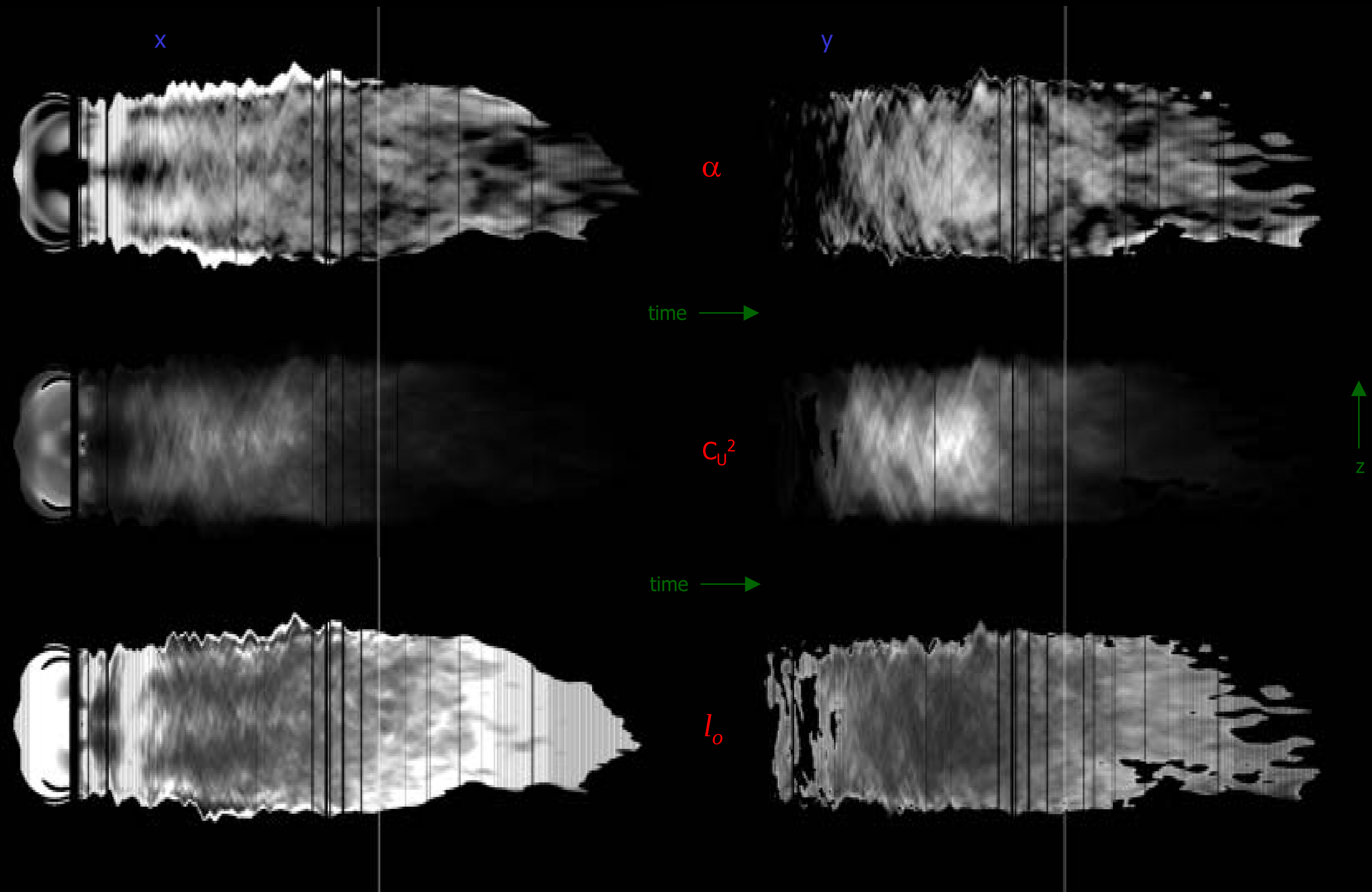
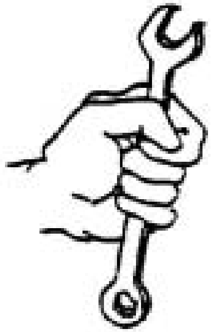


Werne, Meyer, Bizon & Fritts, 2002

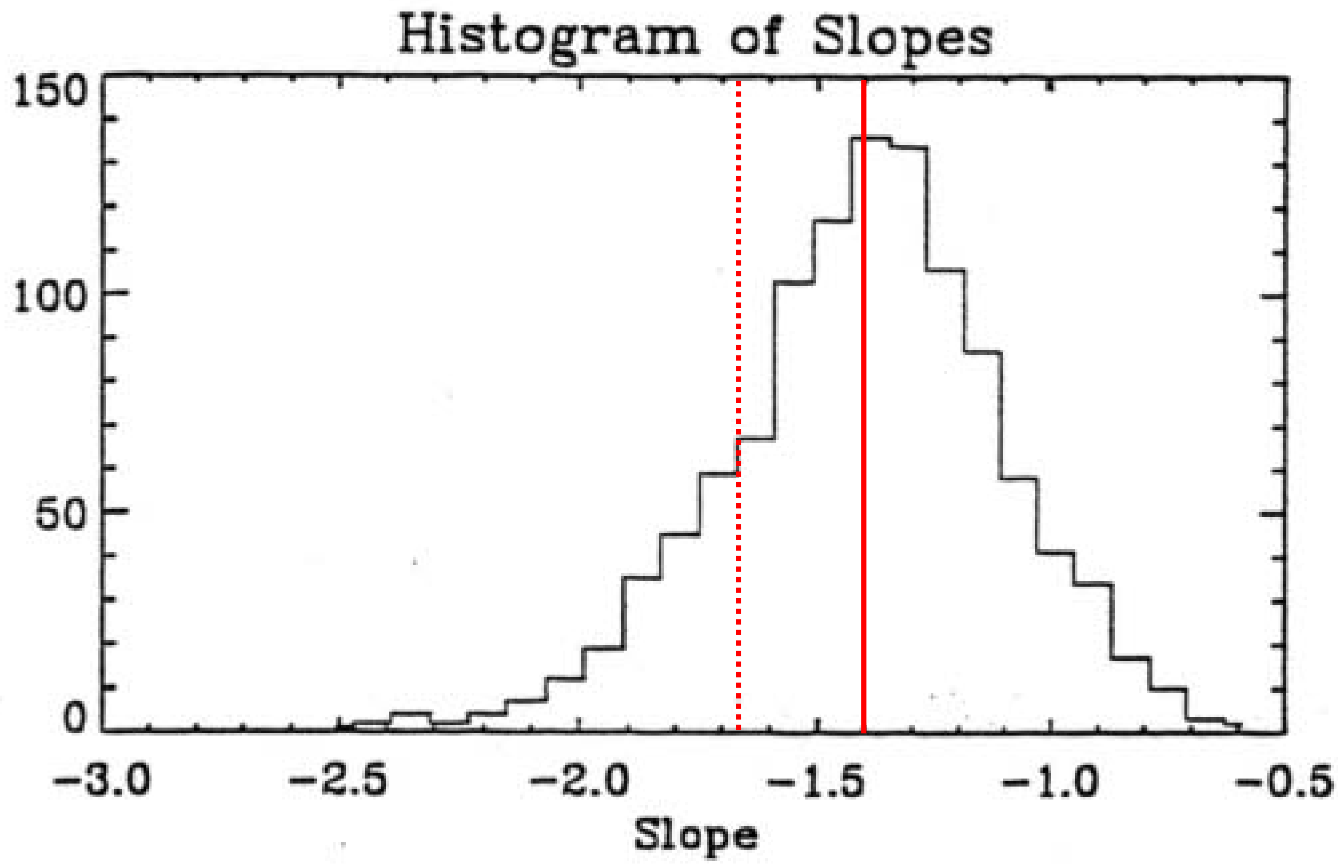



x
y
α
time
C_U^2
z
time
l_o



ABLE ACE anemometry data

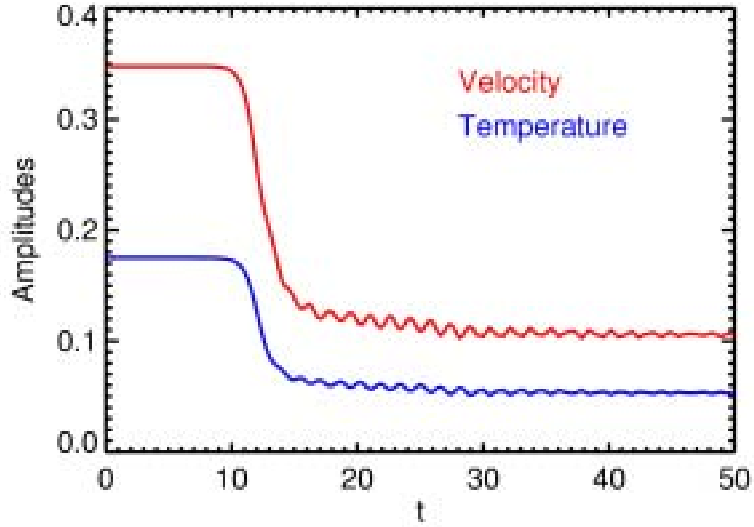
Bruce Masson, 1996

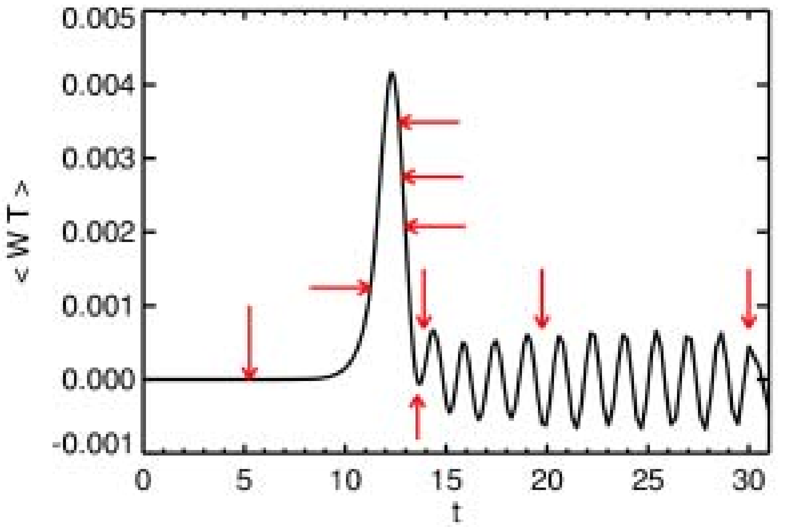


also, Michael Roggeman, private communication, 2001

Gravity Wave: Evolution

Bizon, Werne & Fritts, 2001





Gravity wave: Production and Dissipation

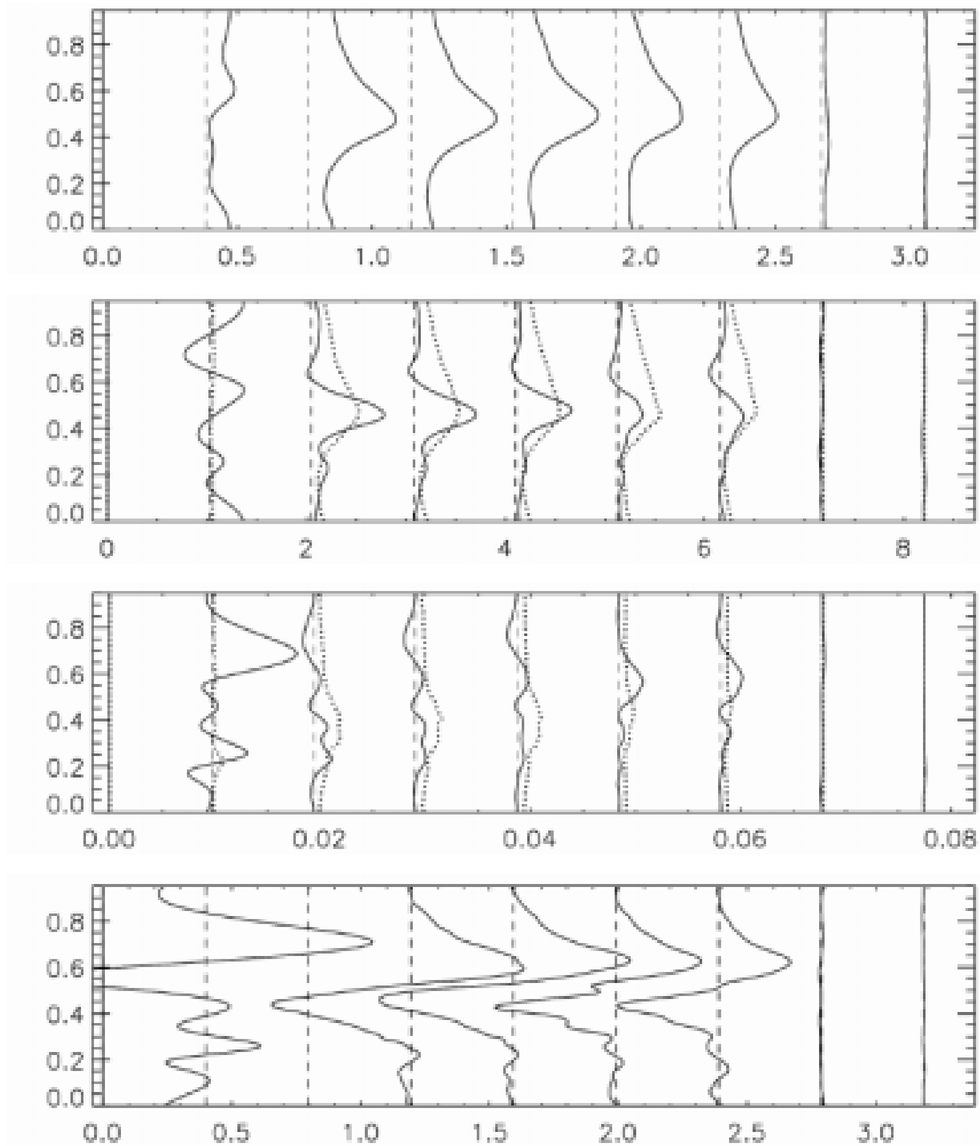
Bizon, Werne & Fritts, 2001

KE

KE prod., ϵ

PE prod., χ

d<pv$_n$>/dn



Conclusions

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7. Breaking gravity waves, in the absence of shear, dissipate rapidly.

8. Gravity-wave breaking is inherently out of balance.

9. Entrainment zones are non-stationary, inhomogeneous, and anisotropic; unfortunately they also have the greatest impact on optical propagation.

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Combine Simulation and Observation for an operational path

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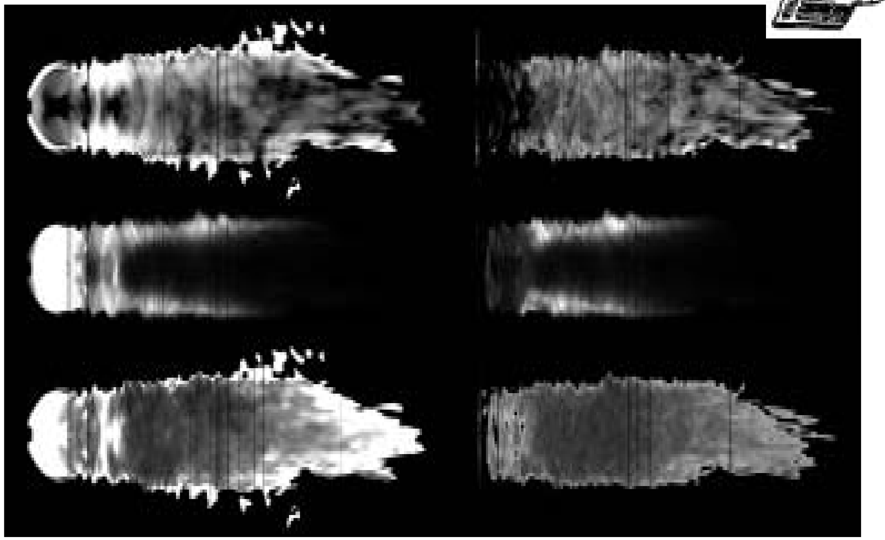

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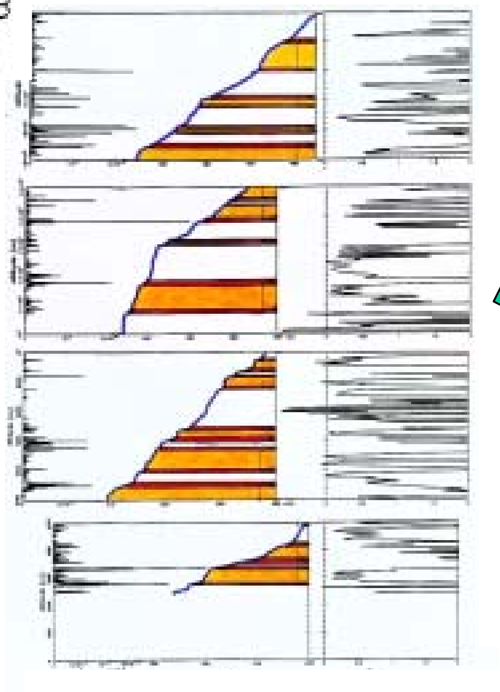
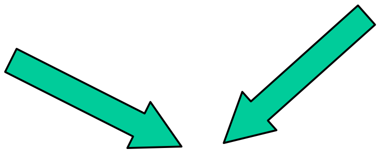
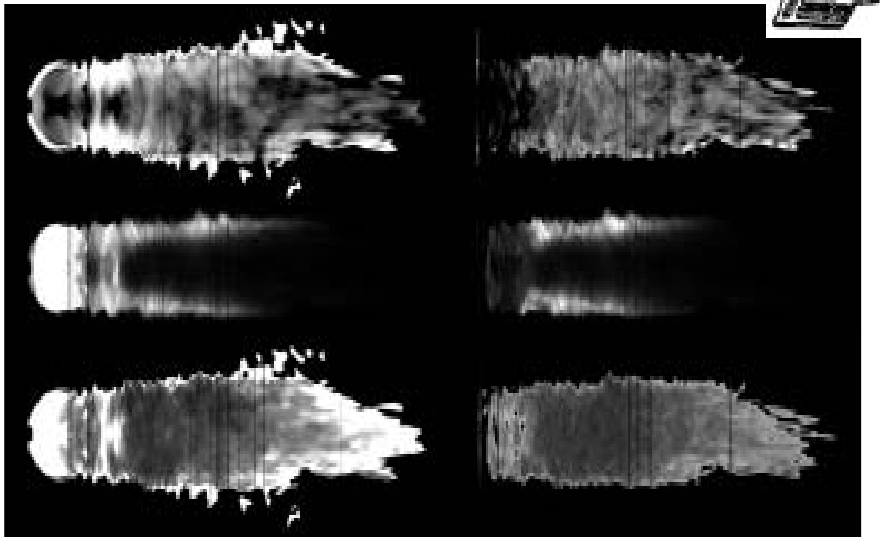
Coulman, Vernin & Fuchs, *Applied Optics* 34 5461 (1995)



C_T^2 T R_i





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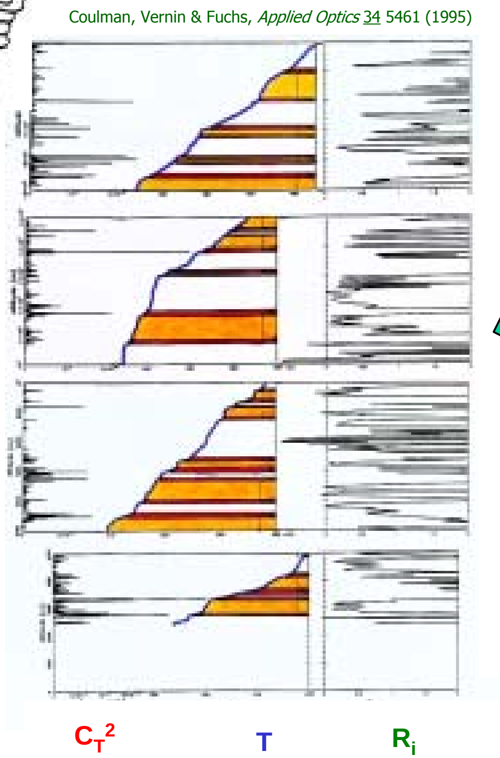

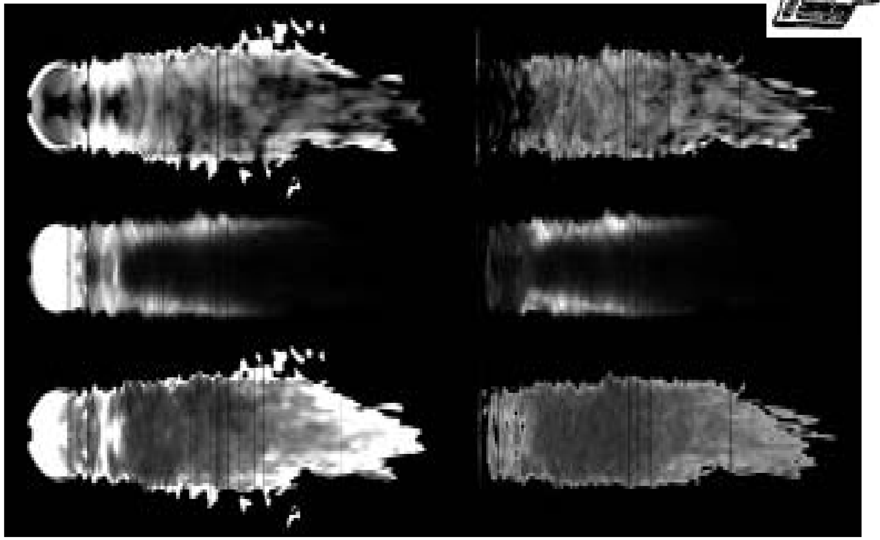

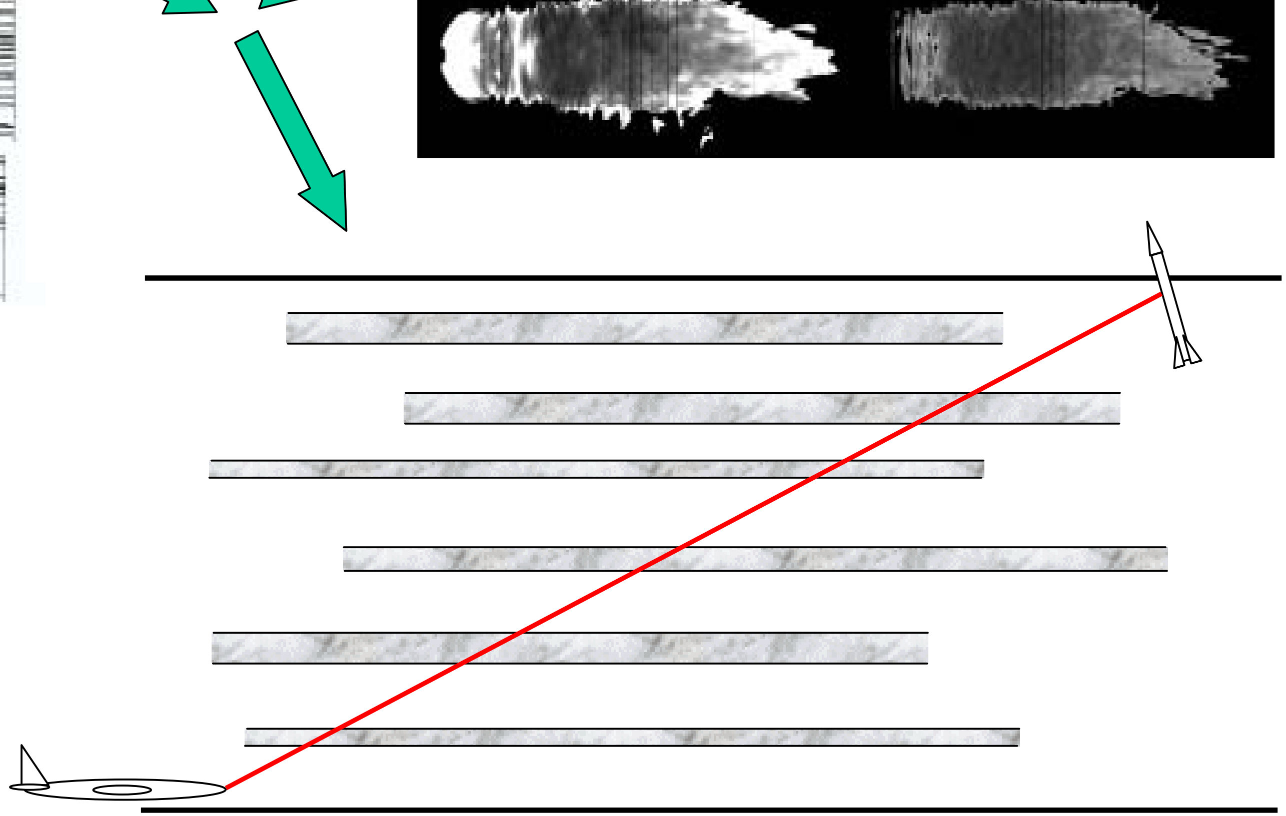

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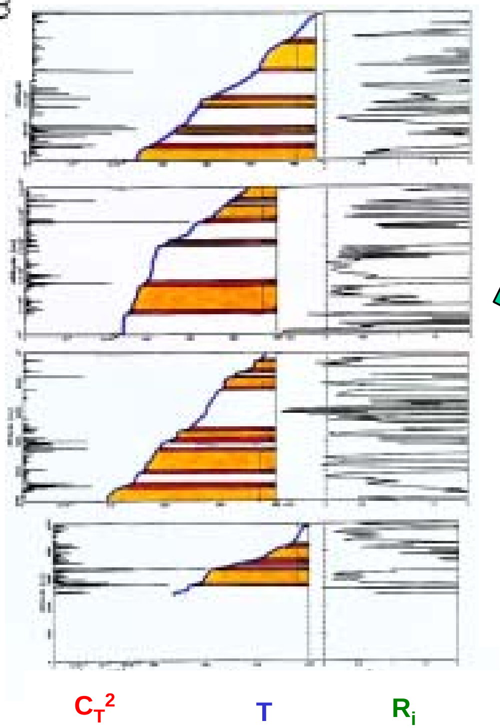

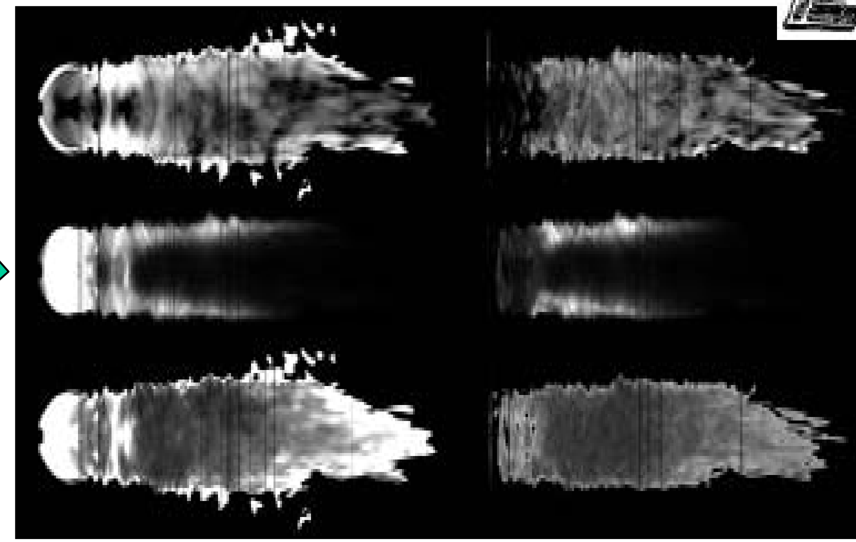

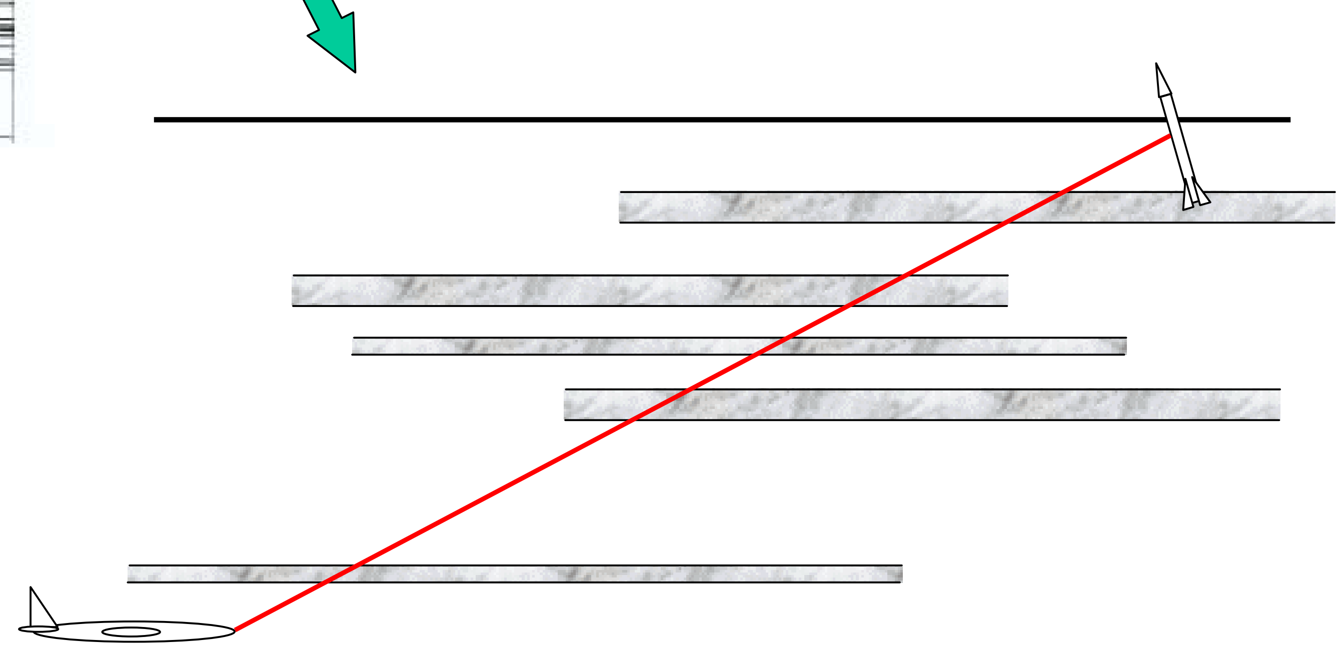

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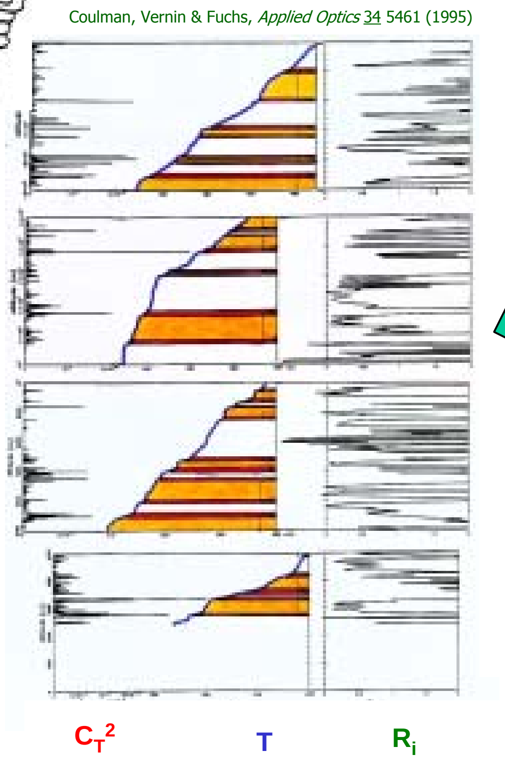

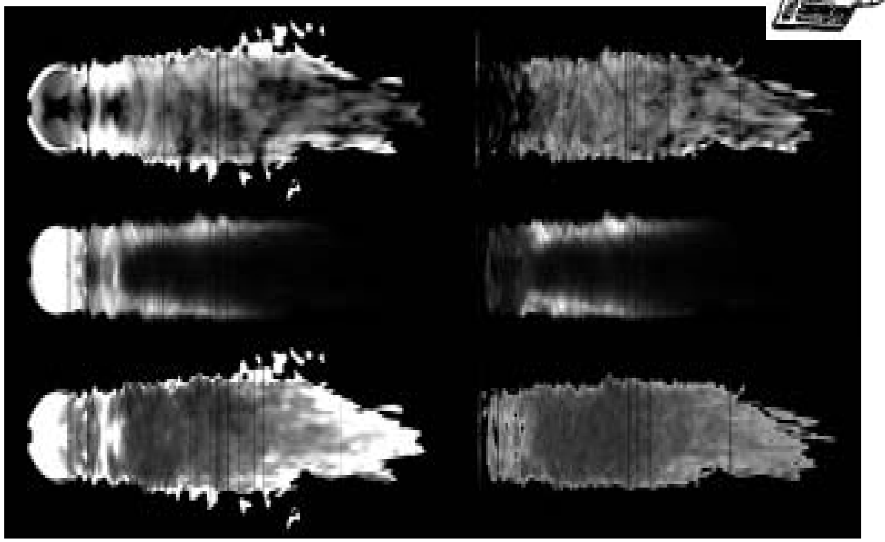

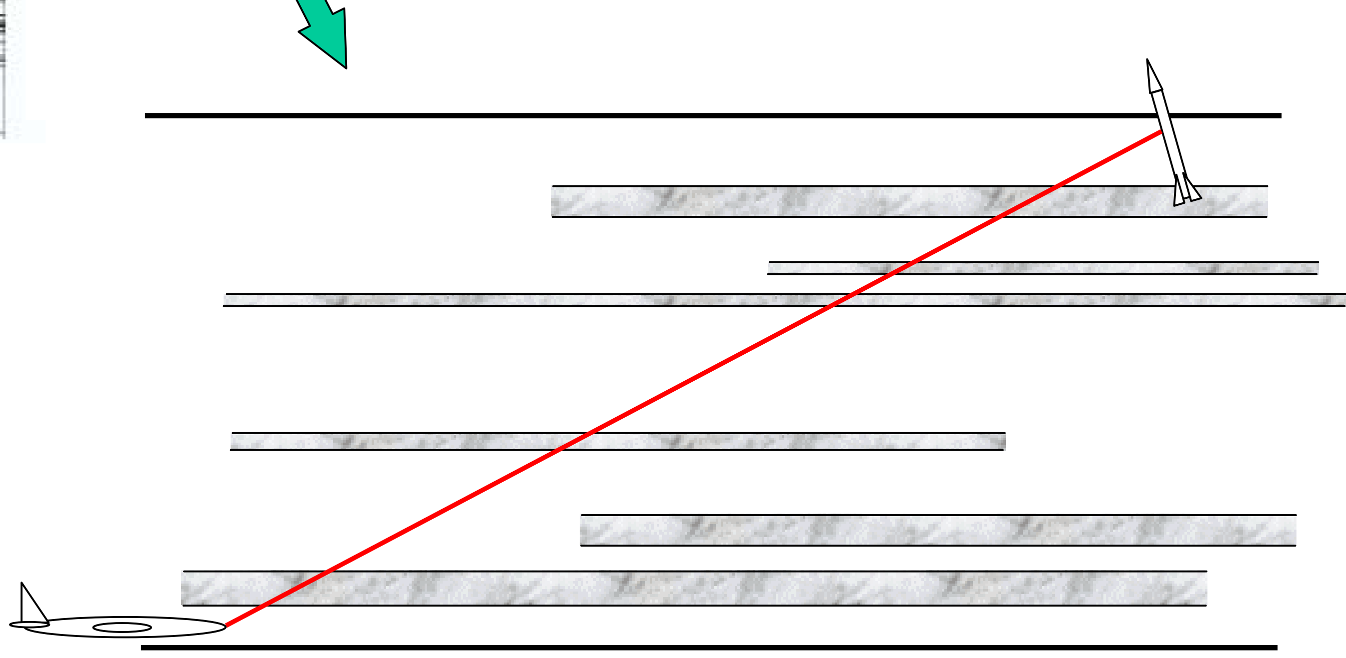

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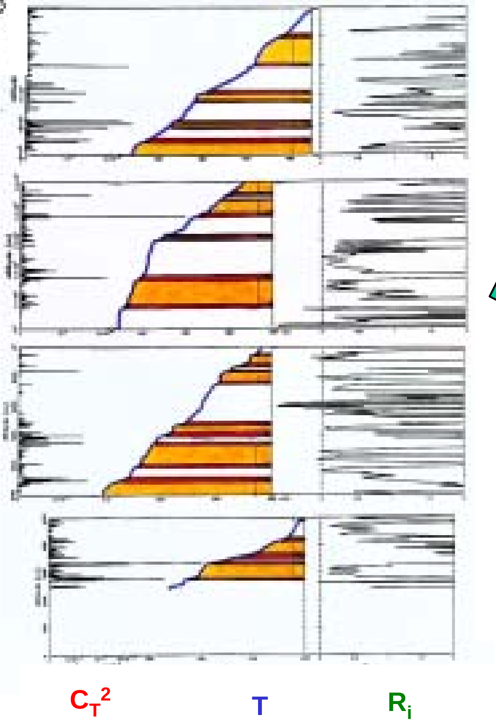

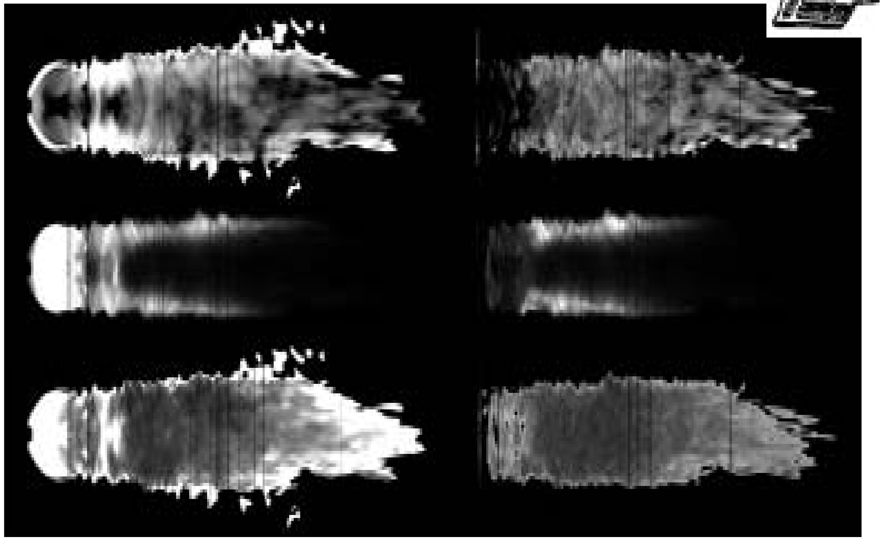

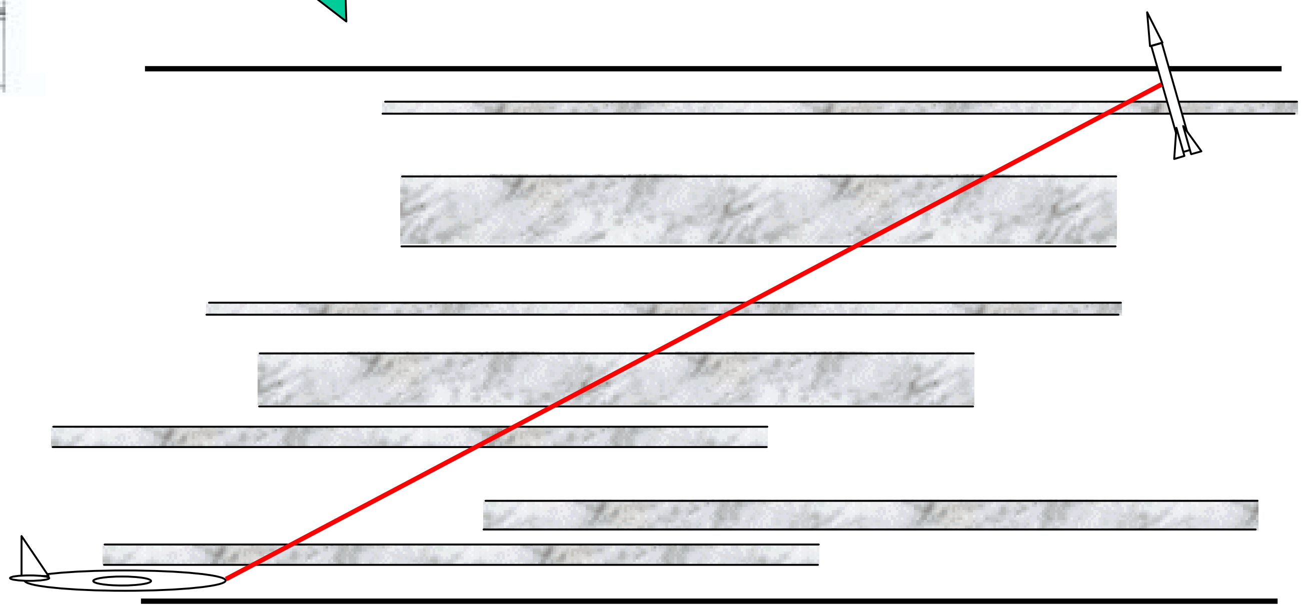

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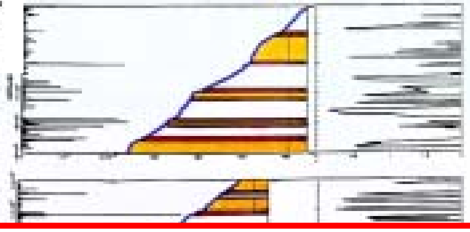

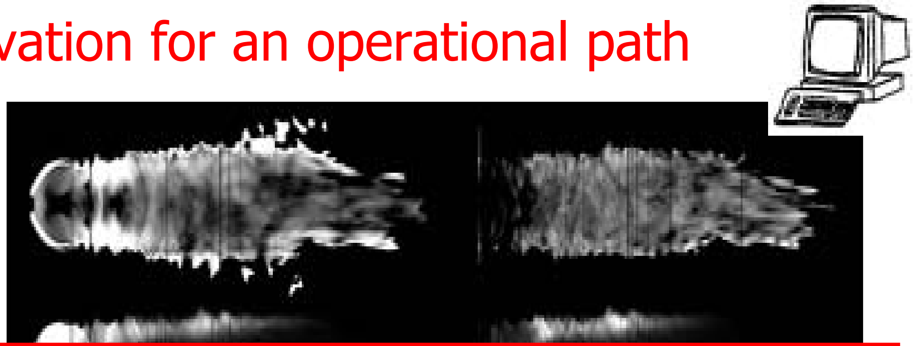

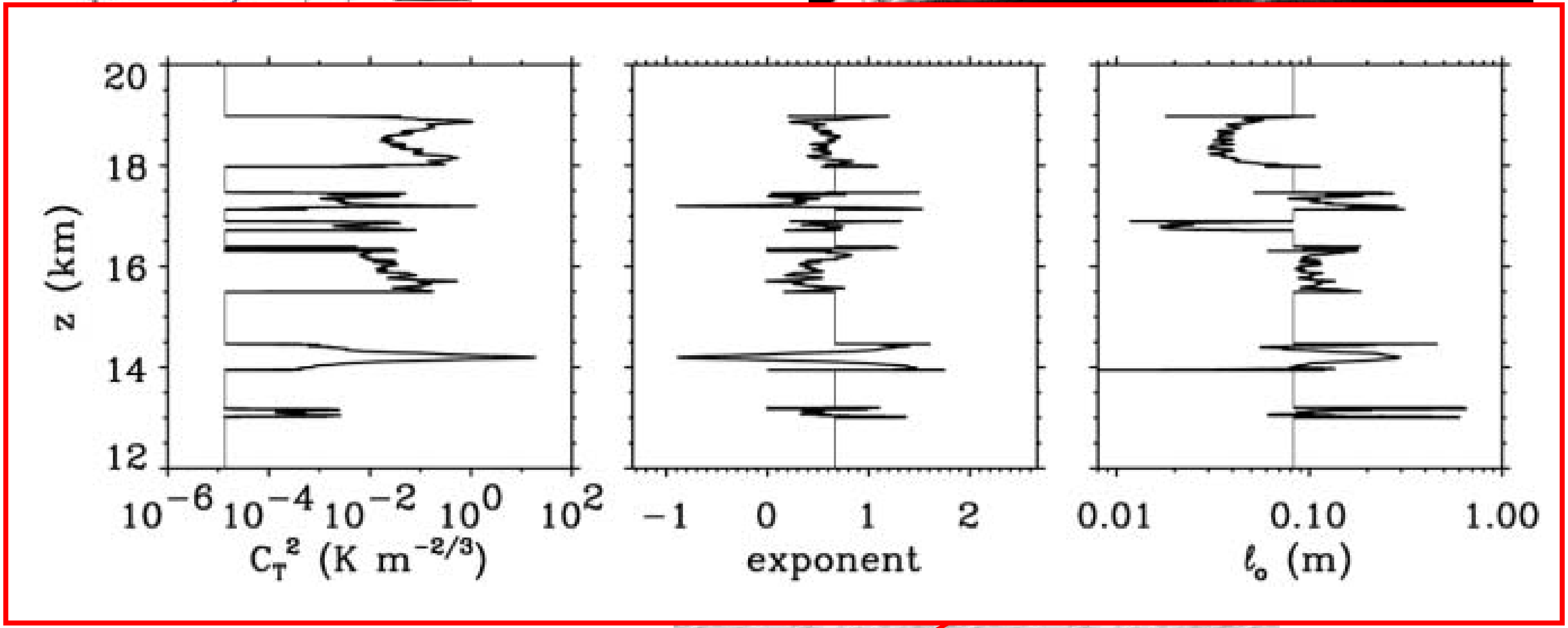

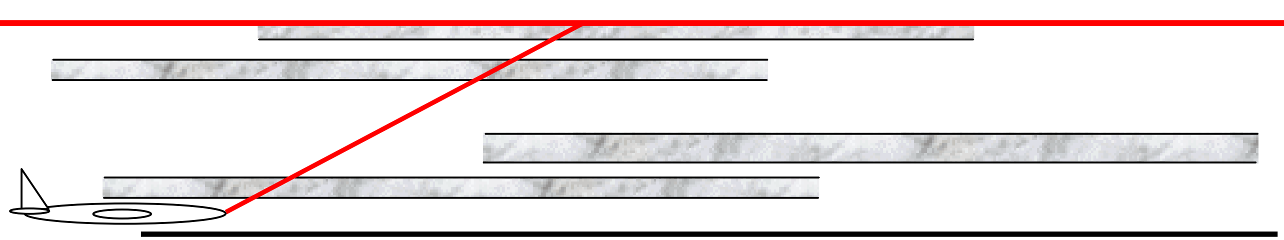

DNS Efforts

- Validate simulations
- Characterize/quantify atmospheric turbulence

Phase Screen Specification

- Combine observation and simulation for long paths
- Quantify non-Kolmogorov effects

Turbulence Simulation Algorithm Development

- SGS parameterizations
- Better upper boundary conditions

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RESOLVED FIELDS AND SFS MOMENTUM FLUXES

$$U_i = \overline{U_i} + u_i \equiv \int U_i(x'_j) G(x_i, x'_j) dx'_j + u_i$$

The SFS fluxes in LES are:

$$\tau_{ij} = \overline{U_i U_j} - \overline{U_i}\;\overline{U_j}$$

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The SFS fluxes in LES are (Germano, 1986):

$$\tau_{ij} = \overline{U_i U_j} - \overline{U_i}\,\overline{U_j} = L_{ij} + C_{ij} + R_{ij}$$

$$\begin{array}{lll}
\text{“Leonard”} & L_{ij} & = \overline{\overline{U_i}\,\overline{U_j}} - \overline{\overline{U_i}}\,\overline{\overline{U_j}} \\
\text{Cross} & C_{ij} & = \overline{\overline{U_i} u_j} + \overline{\overline{U_j} u_i} - \overline{\overline{U_i}}\,\overline{u_j} - \overline{\overline{U_j}}\,\overline{u_i} \\
\text{Reynolds} & R_{ij} & = \overline{u_i u_j} - \overline{u_i}\,\overline{u_j}
\end{array}$$

Galilean invariant

Modeling τ_{ij}

Modeling τ_{ij}

- Eddy viscosity models $\tau_{ij} = -2\nu_t S_{ij}$
  - Smagorinsky $\nu_t = (C_s l)^2 |S|$
  - TKE $\nu_t = C_k l \sqrt{E_s}$
  - length scale $l = \Delta_f$ and $l = f(S, N)$

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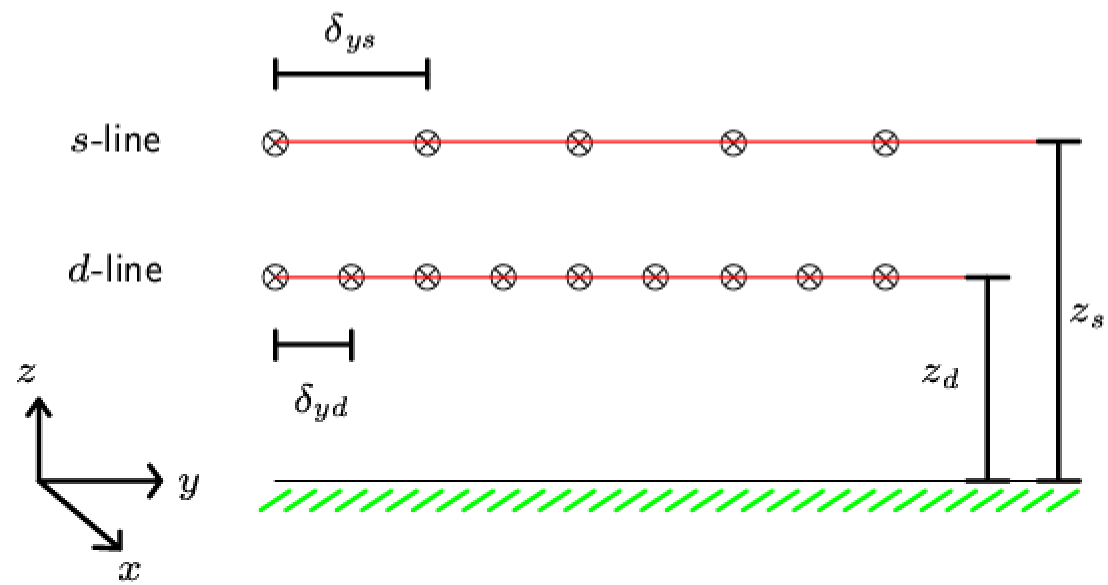
Horizontal Array Turbulence Study (HATS) was designed to test SGS techniques.

HORIZONTAL ARRAY TURBULENCE STUDY (HATS)

- Field campaign to measure $\mathcal{T}_{ij}, \mathcal{T}_{i\theta}$ over a wide range of stratification Horst *et al.* (2002) NCAR, JHU, PSU

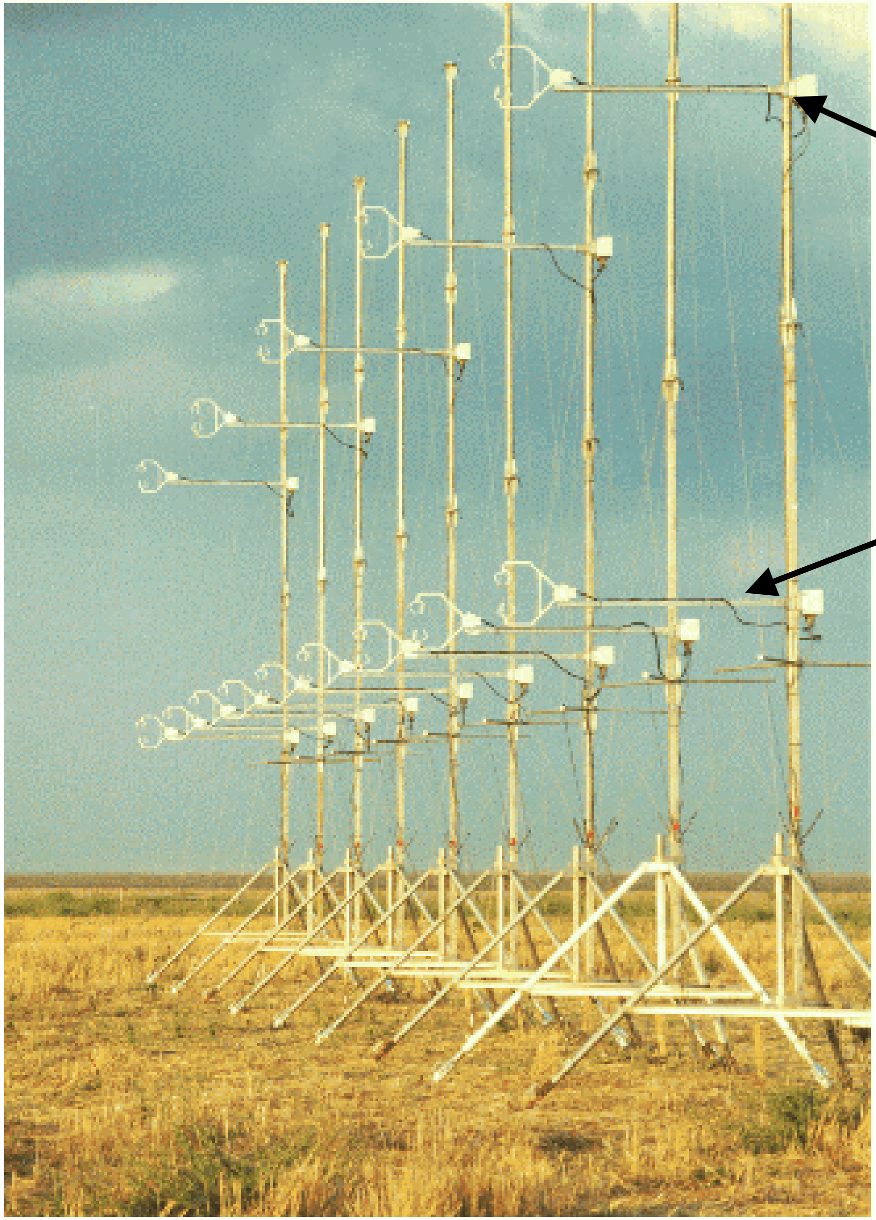
- Based on the horizontal array technique Tong *et al.* (1998), (1999) and Porté-Agel *et al.* (2001)

- 38 cases, 4 different sonic arrays, $-2 < z/L < 2$



HATS data is available from T. Horst, horst@ucar.edu

Horizontal Array Turbulence Study (HATS)



Four different sonic arrays, 38 cases

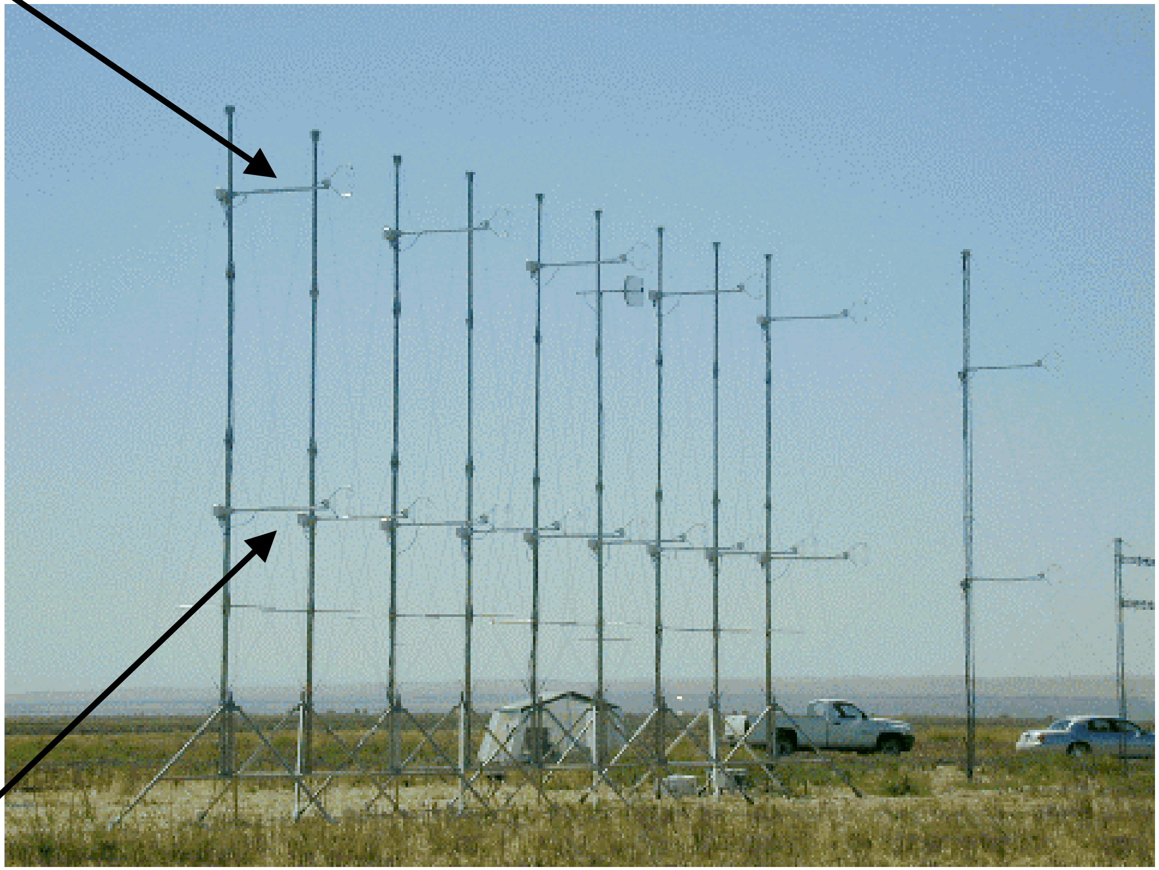
z = 6.90m, dy = 6.70m
z = 8.66m, dy = 4.33m
z = 8.66m, dy = 2.17m
z = 5.15m, dy = 0.63m

z = 3.45m, dy = 3.35m
z = 4.33m, dy = 2.17m
z = 4.33m, dy = 1.08m
z = 4.15m, dy = 0.50m

Tom Horst et al (2002), NCAR, JHU, PSU
Pete Sullivan, NCAR

ARRAY-2

$z_s = 8.66\text{m}, dy_s = 4.33\text{m}$



$z_d = 4.33\text{m}, dy_d = 2.17\text{m}$

ARRAY-3

$z_d = 8.66\text{m}, dy_d = 2.17\text{m}$

$z_s = 4.33\text{m}, dy_s = 1.08\text{m}$

ARRAY-4

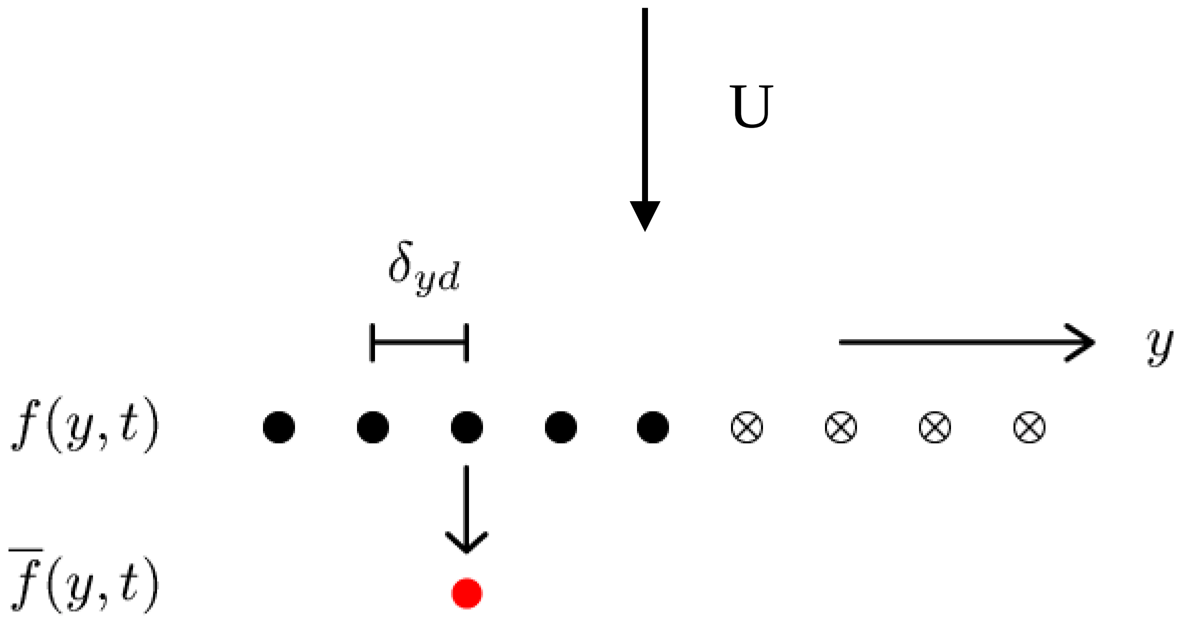
$z_s = 5.15\text{m}$

$dy_s = 0.63\text{m}$

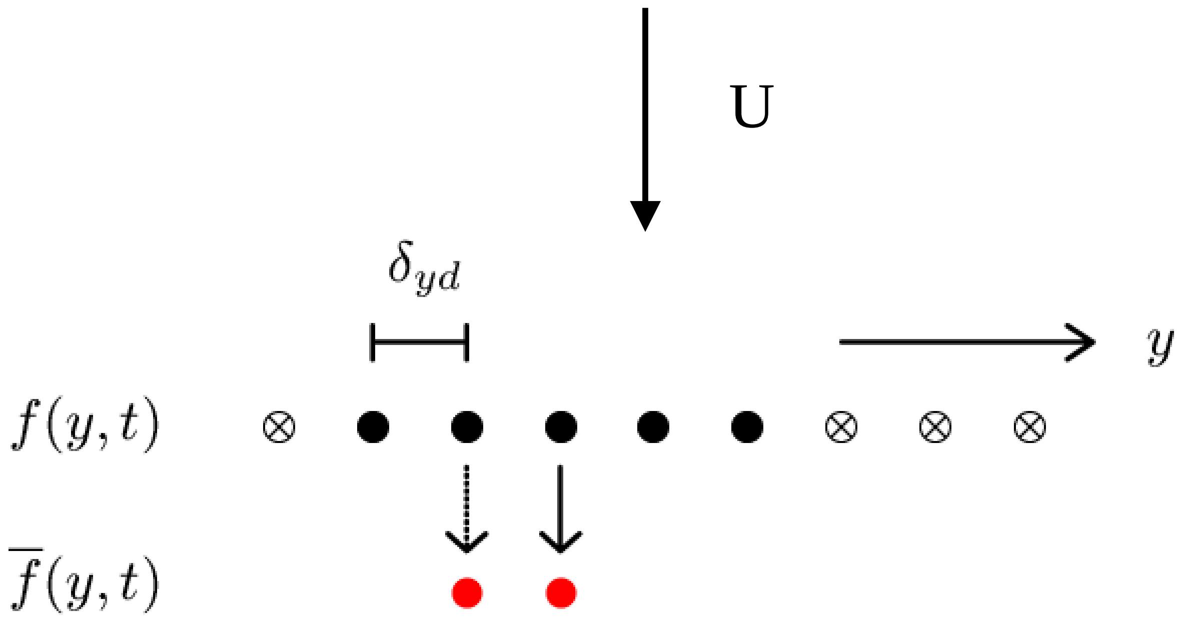
$z_d = 4.15\text{m}$

$dy_d = 0.50\text{m}$

AN EXAMPLE OF LATERAL (Y) FILTERING



AN EXAMPLE OF LATERAL (Y) FILTERING



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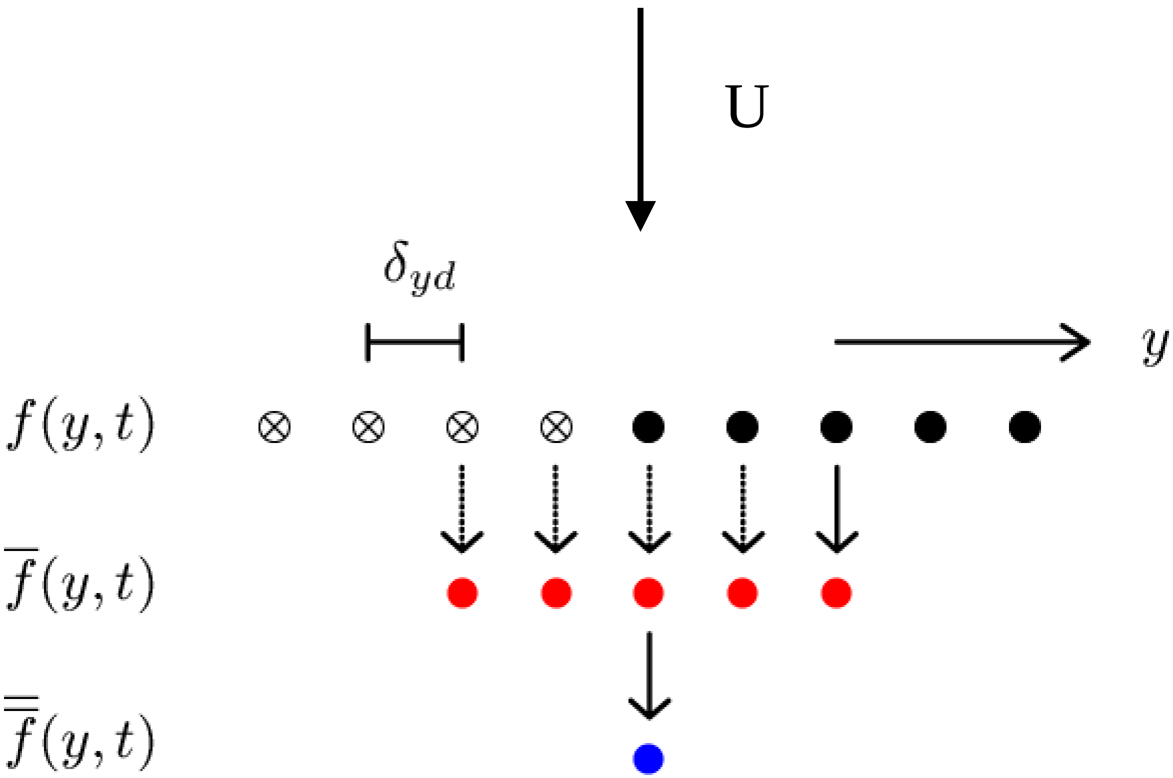
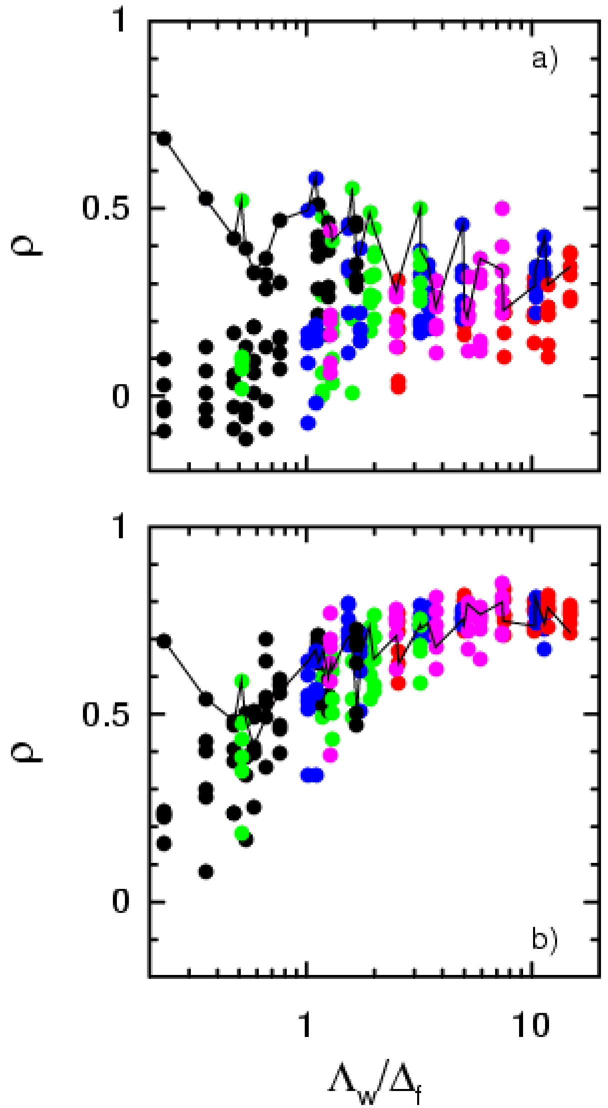


We use top-hat filtering in y and Gaussian filtering in x or t

Model-Data Correlations

without L_{ij}

with L_{ij}



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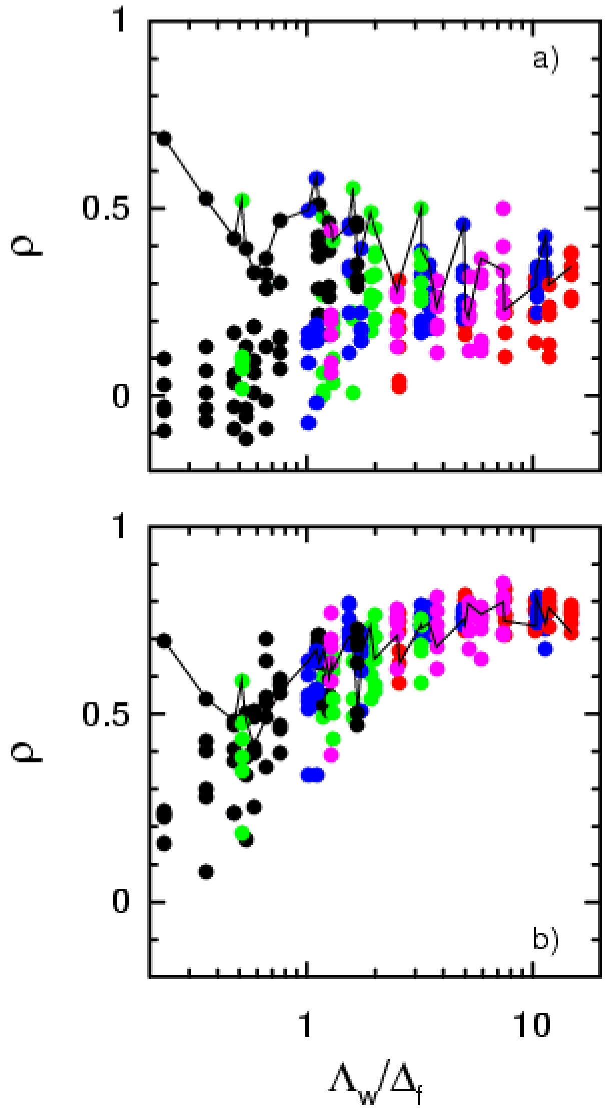

Practical Computation of L_{ij}

$$L_{ij} = \overline{\overline{U_i}\;\overline{U_j}} - \overline{\overline{U_i}}\;\overline{\overline{U_j}}$$

$$\overline{\overline{U_i}\;\overline{U_j}} \approx \overline{U_i}\;\overline{U_j} + \frac{\Delta_f^2}{24}\frac{\partial^2 \overline{U_i}}{\partial x_k \partial x_k} + H.O.T.$$

$$\overline{\overline{U_i}} \approx \overline{U_i} + \frac{\Delta_f^2}{24}\frac{\partial^2 \overline{U_i}}{\partial x_k \partial x_k} + H.O.T.$$

$$L_{ij} \approx \frac{\Delta_f^2}{12}\frac{\partial \overline{U_i}}{\partial x_k}\frac{\partial \overline{U_j}}{\partial x_k}$$

Gravity-Wave Radiation Conditions

Gravity-Wave Radiation Conditions

Linear Boussinesq Equations

$$\begin{aligned} \partial_t u + \partial_x p &= 0 \\ \partial_t w + \partial_z p - Ri\,\theta &= 0 \\ \partial_t \theta + w &= 0 \\ \nabla \cdot \mathbf{u} &= 0 \end{aligned}$$

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$\Rightarrow$

Plane Waves

$$\begin{pmatrix} u \\ w \\ \theta \\ p \end{pmatrix} = \begin{pmatrix} -k_x k_z \\ k_x^2 \\ k_x^2/i\omega \\ -\omega k_z \end{pmatrix} e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$$

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Dispersion Relation

$$\omega^2 / Ri = k_x^2 / k^2$$
$$k_x^2 + k_z^2 = k^2$$

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Elliptic Equation for Pressure
(How should we handle BC's?)

$$\nabla^2 p = Ri\,\partial_z \theta$$

Gravity-Wave Radiation Conditions

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$$\partial_t u + \partial_x p = 0$$

$$\partial_t w + \partial_z p - Ri\,\theta = 0 \quad (1)$$

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Dispersion Relation

$$\omega^2/Ri = k_x^2/k^2 \quad (3)$$

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Elliptic Equation for Pressure (How should we handle BC's?)

$$\nabla^2 p = Ri\,\partial_z \theta$$

$\xrightarrow{1}$

Klemp & Durran (1983): Solve (1) & (2) subject to (3)

$$\left(Ri - \omega^2\right) w = -\omega k_z P$$

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For low frequency …

$$P \approx w\sqrt{Ri}\,/k_x$$

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For low frequency …

$$P \approx w\sqrt{Ri}\,/k_x$$

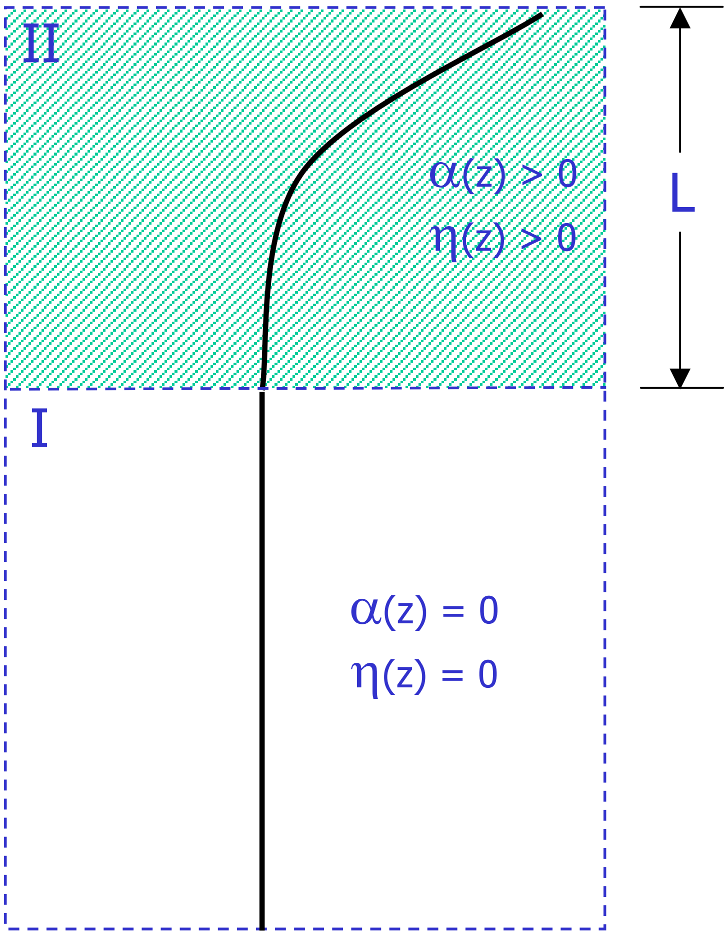
OK if kz>>kx, but other waves are trapped!

Gravity-Wave Radiation Conditions

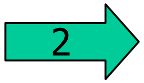
2

Gravity-Wave Radiation Conditions

2



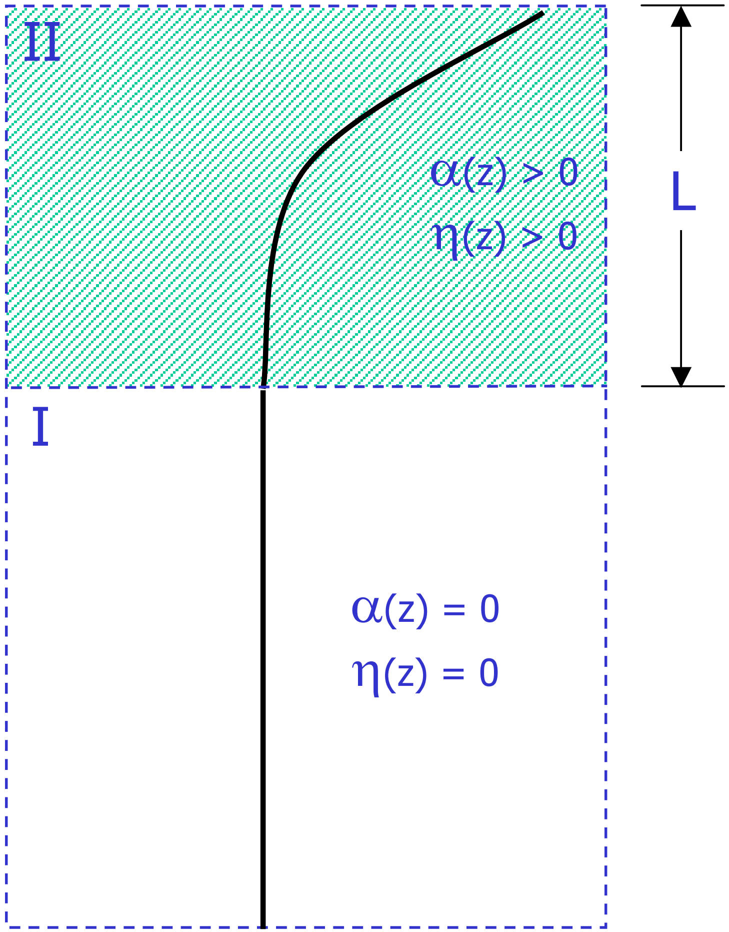
Gravity-Wave Radiation Conditions



Linear Boussinesq Equations

$$[\partial_t + \underline{\alpha(z)}]\, u + \partial_x p = 0$$
$$[\partial_t + \underline{\alpha(z)}]\, w + \partial_z p - Ri\,\theta = 0$$
$$[\partial_t + \underline{\eta(z)}]\, \theta + w = 0$$
$$\nabla \cdot \mathbf{u} = 0$$

$$\nabla^2 p = Ri\,\partial_z\theta - \underline{\alpha(z)}'w$$



Gravity-Wave Radiation Conditions

2

Linear Boussinesq Equations

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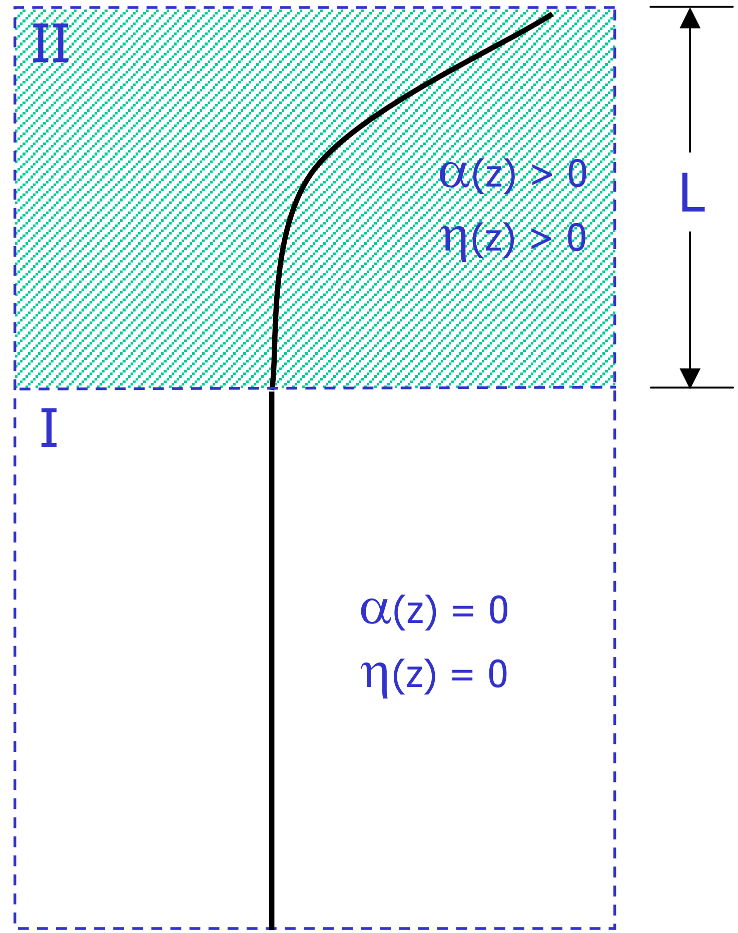

1. If $\alpha(z)$ too small, reflections from outer boundary.
2. If $\alpha(z)$ too large, reflections from inner boundary.
3. If $\lambda > L$, reflect from II ➔ must make L large.

Gravity-Wave Radiation Conditions

Analyze 1 and 2 by recasting as a scattering problem.

Gravity-Wave Radiation Conditions

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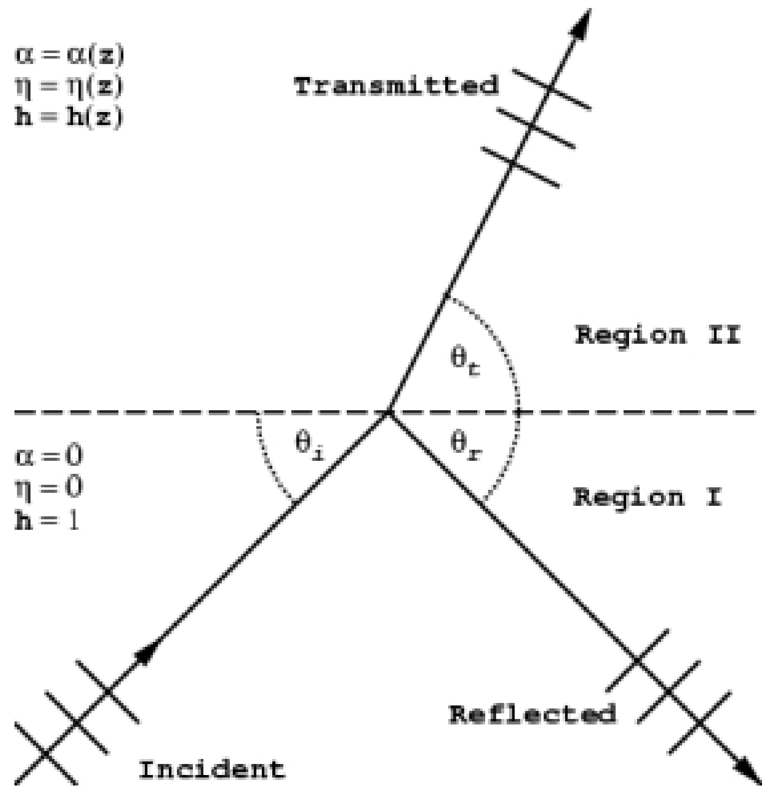
Linear Boussinesq Equations

$$\left[\partial_t + \underline{\alpha(z)}\right] u + \partial_x p = 0$$

$$\left[\underline{h(z)}\partial_t + \underline{\alpha(z)}\right] w + \partial_z p - Ri\,\theta = 0$$

$$\left[\partial_t + \underline{\eta(z)}\right] \theta + w = 0$$

$$\nabla \cdot \mathbf{u} = 0$$



Gravity-Wave Radiation Conditions

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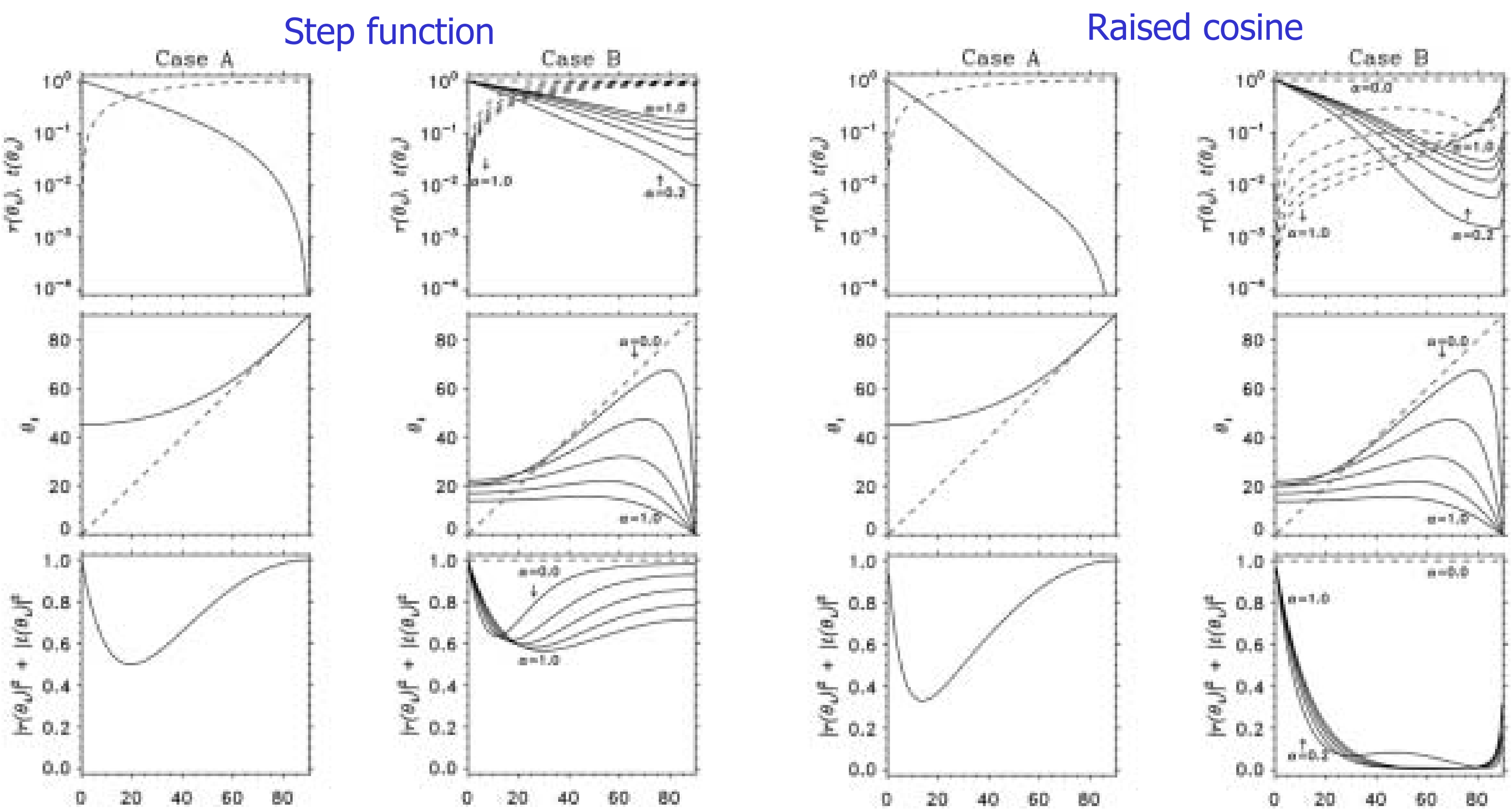

1. Reflection is reduced at high θ.

2. Smooth variation decreases reflection.

Gravity-Wave Radiation Conditions

3

Gravity-Wave Radiation Conditions

3 Try a different approach: Start with the solution you want.

Gravity-Wave Radiation Conditions

3

Try a different approach: Start with the solution you want.

Damped Plane Waves

$$\begin{pmatrix} u \\ w \\ \theta \\ p \end{pmatrix} = \begin{pmatrix} -k_x k_z + \underline{i\beta(z) k_x k_z/\omega} \\ k_x^2 \\ k_x^2/i\omega \\ -\omega k_z \end{pmatrix} e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)+\underline{(k_z/\omega)\int_0^z \beta(s)ds}}$$

Gravity-Wave Radiation Conditions

3 Try a different approach: Start with the solution you want.

Linear Boussinesq Equations

$$\begin{aligned} \partial_t u + \partial_x p &= 0 \\ \partial_t w + \partial_z p - Ri\,\theta &= 0 \\ \partial_t \theta + w &= 0 \\ \nabla \cdot \mathbf{u} &= 0 \end{aligned}$$

Damped Plane Waves

$$\begin{pmatrix} u \\ w \\ \theta \\ p \end{pmatrix} = \begin{pmatrix} -k_x k_z + \underline{i\beta(z) k_x k_z/\omega} \\ k_x^2 \\ k_x^2/i\omega \\ -\omega k_z \end{pmatrix} e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)+\underline{(k_z/\omega)\int_0^z \beta(s)ds}}$$

Dispersion Relation

$$\omega^2/Ri = k_x^2/k^2$$

Gravity-Wave Radiation Conditions

3 Try a different approach: Start with the solution you want.

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Dispersion Relation

$$\omega^2/Ri = k_x^2/k^2$$

$$\begin{aligned}\partial_t u + \partial_x p &= -U_0\beta\\ \partial_t w + \partial_z p - Ri\,\theta &= -W_0\beta\\ \partial_t \theta + w &= 0\\ \nabla \cdot \mathbf{u} &= 0\end{aligned}$$

where

$$\begin{aligned}U_0 &= \hat{p} k_x/\omega\\ W_0 &= -\hat{p} k_z/\omega\end{aligned}$$

Gravity-Wave Radiation Conditions

3 Try a different approach: Start with the solution you want.

Linear Boussinesq Equations

$$\begin{aligned} \partial_t u + \partial_x p &= 0 \\ \partial_t w + \partial_z p - Ri\,\theta &= 0 \\ \partial_t \theta + w &= 0 \\ \nabla \cdot \mathbf{u} &= 0 \end{aligned}$$

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where

$$\begin{aligned} U_0 &= \hat{p} k_x/\omega \\ W_0 &= -\hat{p} k_z/\omega \end{aligned}$$

$$\begin{aligned} \partial_t U_0 &= -\partial_x \hat{p} \\ (\partial_t - \beta)\, W_0 &= \partial_z \hat{p} \end{aligned}$$

Gravity-Wave Radiation Conditions

3 Try a different approach: Start with the solution you want.

Linear Boussinesq Equations

$$\partial_t u + \partial_x p = 0$$
$$\partial_t w + \partial_z p - Ri\,\theta = 0$$
$$\partial_t \theta + w = 0$$
$$\nabla \cdot \mathbf{u} = 0$$

Damped Plane Waves

$$\begin{pmatrix} u \\ w \\ \theta \\ p \end{pmatrix} = \begin{pmatrix} -k_x k_z + \underline{i\beta(z) k_x k_z/\omega} \\ k_x^2 \\ k_x^2/i\omega \\ -\omega k_z \end{pmatrix} e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)+\underline{(k_z/\omega)\int_0^z \beta(s)ds}}$$

Dispersion Relation

$$\omega^2/Ri = k_x^2/k^2$$

New System of Equations for Damping Layer (PML)

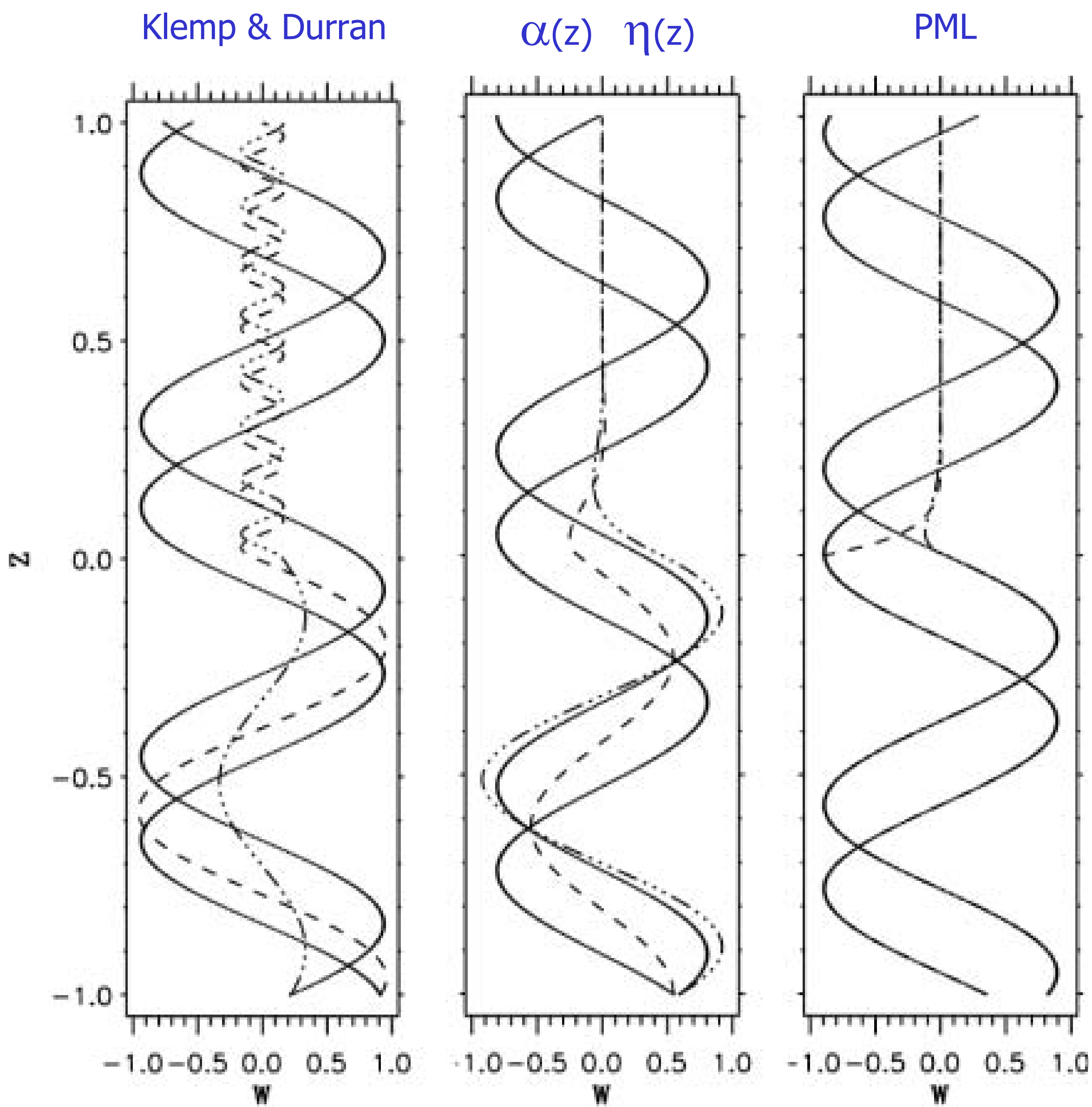
$$\boxed{\begin{aligned} \partial_t u + \partial_x p &= -U_0\beta \\ \partial_t w + \partial_z p - Ri\,\theta &= -W_0\beta \\ \partial_t \theta + w &= 0 \\ \nabla \cdot \mathbf{u} &= 0 \end{aligned}}$$

where

$$U_0 = \hat{p} k_x/\omega$$
$$W_0 = -\hat{p} k_z/\omega$$

$$\boxed{\begin{aligned} \partial_t U_0 &= -\partial_x \hat{p} \\ (\partial_t - \beta)\, W_0 &= \partial_z \hat{p} \end{aligned}}$$

Gravity-Wave Radiation Conditions



Gravity-Wave Radiation Conditions

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PML at finite Re

$$\left(\partial_t - Re^{-1}\nabla^2\right) u + \partial_x p = -U_0\beta - Re^{-1}\partial_z\left(2\partial_z U_6\beta - U_5\beta^2 + U_6\beta'\right)$$

$$\left(\partial_t - Re^{-1}\nabla^2\right) w + \partial_z p - Ri\,\theta = -W_0\beta + Re^{-1}\partial_x\left(2\partial_z U_6\beta - U_5\beta^2 + U_6\beta'\right)$$

$$\left(\partial_t - Pe^{-1}\nabla^2\right)\theta + w = -Pe^{-1}\left(2\partial_z\theta_3\beta - \theta_2\beta^2 + \theta_3\beta'\right)$$

$$\nabla\cdot\mathbf{u} = 0$$

$$\partial_t U_0 = -\partial_x p$$

$$\left(\partial_t - \beta\right) W_0 = \partial_z p$$

$$\left(\partial_t - \beta\right) U_6 = -u$$

$$\left(\partial_t - \beta\right) U_5 = -\partial_z U_6$$

$$\left(\partial_t - \beta\right)\theta_3 = -\partial_z\theta$$

$$\left(\partial_t - \beta\right)\theta_2 = -\partial_z\theta_3$$

Gravity-Wave Radiation Conditions

References

1. Klemp & Durran, 1983: An upper boundary condition permitting internal gravity wave radiation in numerical mesoscale models, *Monthly Weather Review* **111**, 430-444.

2. Abarbanel, Gottlieb & Hesthaven, 1999: Well-posed Perfectly Matched Layers for Advective Acoustics, *JCP* **154**, 266-283.

3. Tam, Auriault & Cambuli, 1998: Perfectly Matched Layer as an Absorbing Boundary Condition for the Linearized Euler Equations in Open and Ducted Domains, *JCP* **144**, 213-234.

4. Hesthaven, 1998: On the Analysis and Construction of Perfectly Matched Layers for the Linearized Euler Equations, *JCP* **142**, 129-147.

5. Collino & Monk, 1998: The Perfectly Matched Layer in Curvilinear Coordinates, *Siam J. Sci. Comput.* **19**, 2061-2090.

6. Hu, 1996: On Absorbing Boundary Conditions for Linearized Euler Equations by a Perfectly Matched Layer, *JCP* **129**, 201-219.

ABL Future Work Wish List

- Continued comparison with data
  - → CASES-99
  - → VTMX
  - → Air Force Balloon and Radar
- *A priori* tests and SGS development
  - → Eddy-viscosity models
  - → Velocity-estimation models
  - Event catalog for meso-scale models
- Characterize nature of stratified turbulence
  - → Turbulence/billow/mean-flow
  - → Stability profile
  - → Turbulence anisotropy
- Investigate impact of initial conditions
  - Amplitude and shape of noise spectrum
  - → Optimal perturbations
  - Vary Ri and nonlinear thermal structure
- High-resolution wave-breaking solutions
  - → High-Re incompressible solutions
- Spatial modulation and distribution of turbulence
  - Multiple billow/wave interactions
  - → Phase-screen specification using combined DNS/observation results

ABL-Related Publications

- Chen, Kelley, Gibson-Wilde, Werne & Beland, 2001: "Comparison of observed lower atmospheric turbulent structures with a direct numerical simulation" *Annales Geophysicae*, (in press).
- Dubrulle, Laval, Sullivan & Werne, 2001: "A new dynamical subgrid model for the planetary surface layer. I. The model and a priori tests" *J. Atmos. Sci.* (in press).
- Werne, Bizon, Meyer & Fritts, 2001: "Wave-breaking and shear turbulence simulations in support of the Airborne Laser" 11th DoD UGC, Biloxi.
- Werne & Fritts, 2001: "Anisotropy in a stratified shear layer" *Physics and Chemistry of the Earth*, **26**, 263.
- Werne, Adams & Sanders, 2001: "Hierarchical Data Structure and Massively Parallel I/O" *Parallel Computing* (submitted).
- Werne & Fritts, 2000: "Structure Functions in Stratified Shear Turbulence" 10th DoD HPC UGC, Albuquerque.
- Fritts & Werne, 2000: "Turbulence Dynamics and Mixing due to Gravity Waves in the Lower and Middle Atmosphere" in Atmospheric Science across the Stratopause, Geophysical Monograph 123, American Geophys. Union, 143-159.
- Gibson-Wilde, Wene, Fritts & Hill, 2000: "Direct numerical simulation of VHF radar measurements of turbulence in the mesosphere" *Radio Sci.* **35**, 783.
- Hill, Gibson-Wilde, Werne & Fritts, 1999: "Turbulence-induced fluctuations in ionization and application to PMSE" *Earth Planets Space*, **51**, 499.
- Werne & Fritts, 1999: "Stratified shear turbulence: Evolution and statistics" *Geophys. Res. Lett.*, **26**, 439.
- Werne & Fritts, 1999: "Anisotropy in Stratified Shear Turbulence" 9th DoD HPC UGC, Monterey.
- Werne & Fritts, 1998: "Turbulence in Stratified and Sheared Fluids: T3E Simulations" 8th DoD HPC UGC, Houston.